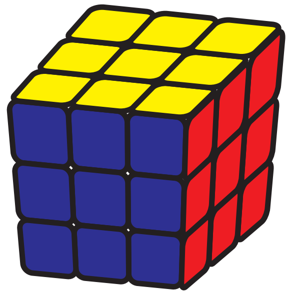
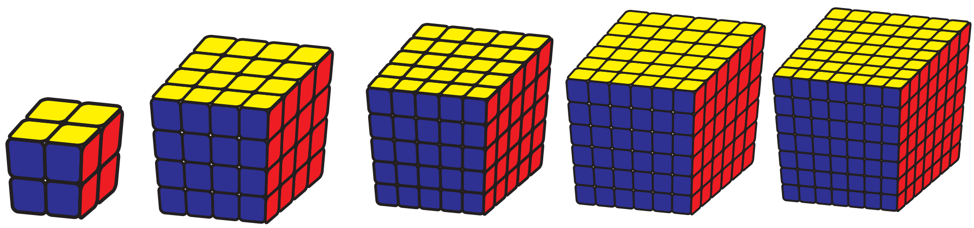
Solutions to the Rotational Cube Puzzles



Introduction

The solutions for all the Rotational Cube Puzzles in production today are provided in this chapter. We begin with the most common puzzle, the 3×3×3, and then cover those of other sizes, from the 2×2×2 up to the 7×7×7.



The solution for the 3×3×3 is thoroughly explained with accompanying illustrations provided as necessary. We have designed the solution instructions to enable a beginner who has never solved a Rotational Cube Puzzle before (perhaps you) to do so successfully. In addition, the solutions will provide an important foundation as well as an introduction to the terms, notation, and move sequences that will be built upon in the larger and more complex cube puzzles.

As you solve the 3×3×3 cube you will learn sequences of moves that are useful for all cubes. These sequences will be shown in detailed diagrams for the smaller cubes with fewer diagrams as you progress to the larger cubes.
The 4×4×4 provides additional challenges and after mastering it, the remaining cube puzzles, the 5×5×5, 6×6×6, and 7×7×7, make use of the same basic sequences.

Advanced Notes:
Small bits of information that may be of interest to those who want to know more about cube-solving will appear in boxes like this one. They will never be necessary to the solution but may help you understand things better.

Each cube is made up of small indivisible cubes called **cubies**. (Technically a cubies is not really a cube, but a piece of plastic that appears to be a cube on the outside surface.) As the puzzle gets larger, the number of cubies increases. The following table shows how many cubies on each cube are movable and therefore must be placed and oriented properly to solve each of the Rotational Cube Puzzles.

Cube Size	Movable Cubies
2×2×2	8
3×3×3	20*
4×4×4	56
5×5×5	92*
6×6×6	152
7×7×7	212*

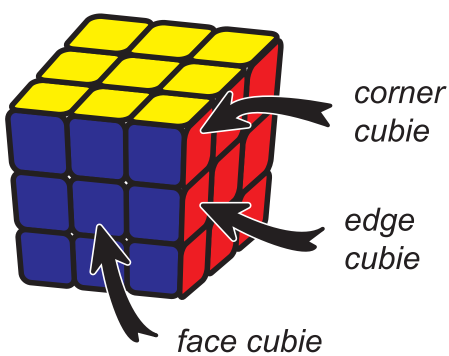
***Advanced Notes:**
For the cubes with an add number of cubies on an edge, the number at left doesn't include the 6 face center cubies, as some experimentation will show you that they can never move (relative to each other). However, if your cube has pictures on the faces, the orientation of these face cubies can matter. We'll show you how to deal with that in a later Advanced Note.

Terminology

The following terms are very important for understanding and solving Rotational Cube Puzzles. They are used throughout our solutions (and by many English-speaking cube solvers around the world) so it is important that you become familiar with them before attempting to follow our instructions.

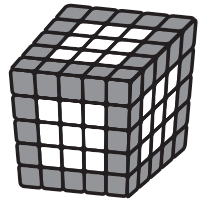
The Cube: The Rotational Cube Puzzle. The Cube is composed of **layers** that can be **turned**. Each layer is made up of **cubies**. The Cube can be a 2×2×2, 3×3×3, 4×4×4, or more.

Cubie: The smallest external part of The Cube. A cubie can have three stickers (**corner cubie**), two stickers (**edge cubie**), or one sticker (**face cubie**) of various colors attached to it. The 3×3×3, for example, has 26 cubies (eight corners, twelve edges, and six faces).

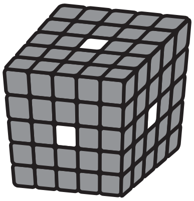


(The diagrams highlight the defined cubies in white.)

Face cubie: A cubie that is on the face of the cube. It has only one color. The 3×3×3 only has one face cubie on each face, and they are also **center cubies**.



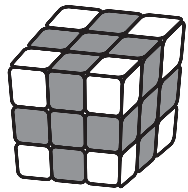
5×5×5 face cubies



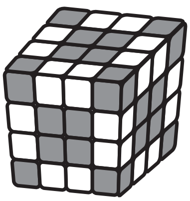
5×5×5 centers

Center cubie: A face cubie that is at the center of the face. They only exist on odd-sized Cubes. Since the center cubies never move relative to each other, they can always be used to determine the correct color arrangement even when the rest of the cube is messed up.

Corner cubie: A cubie that is on a corner of The Cube. It has three differently-colored stickers. All of the Cube puzzles have 8 corner cubies.



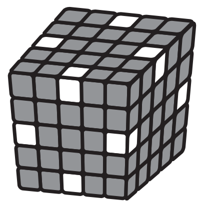
3×3×3 corners



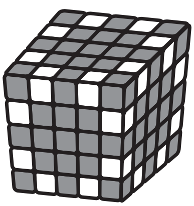
4×4×4 edges

Edge cubie: A cubie that is on an edge of the cube
It has two differently-colored stickers.
The 3×3×3 Cube has 12 edge cubies, the 2×2×2 has *none*, and the larger Cubes have more.

Middle edge cubie: An edge cubie that is on the midpoint of the edge. On the 3×3×3 Cube, all edge cubies are middle edge cubies, so we don't bother using this term.



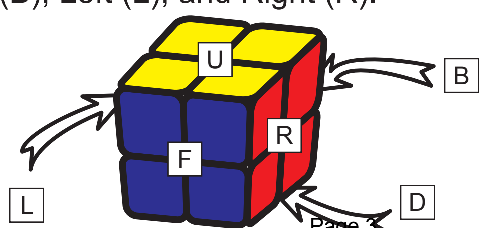
5×5×5 middle edges



5×5×5 side edges

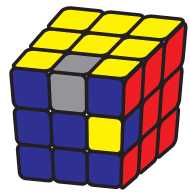
Side edge cubie: An edge cubie that is not a middle edge cubie. On even-sized Cubes such as the 4×4×4 Cube, all edge cubies are side edge cubies.

Face: One of six sides of The Cube. We call the six faces of The Cube Up (U), Down (D), Front (F), Back (B), Left (L), and Right (R). When The Cube is solved, all of the sides of the cubies that show on each face are the same color (or part of a picture). One common mistake is to think "B" stands for "Bottom" -- don't do that!



the six faces

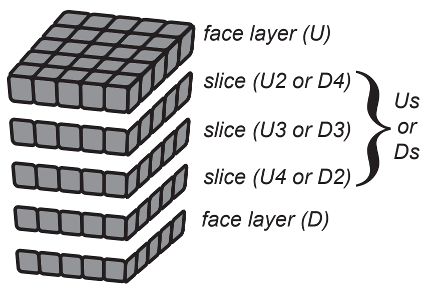
Naming Cubies vs. Naming Positions: A **position** is a location for a cubie, usually designated by the faces the cubie is on Usually this is based on the parts of the cubie that are already solved. For example, the “UF position” (or “UF edge”) is the location of the edge cubie that is on the Up and Front face. The “UF cubie,” on the other hand, refers to the cubie that should belong in the UF position when the cube is solved -- even if the cubie is currently at a different location!



In the diagram above, the UF edge cubie is in the FR position.

> **Advanced Notes:**
> (Some books are careful with the order of letters in naming their positions and cubies to be careful about twists and flips (see next page); for example, “UFR position” and “FRU position” are the same position but with a different cubie orientation. We won’t bother with this distinction.)

Layer: The smallest group of cubies that can be moved together. The layers on the end (that contain an entire face) are often named after the corresponding face (e.g., “The U layer”), and the other layers are called **slices**.

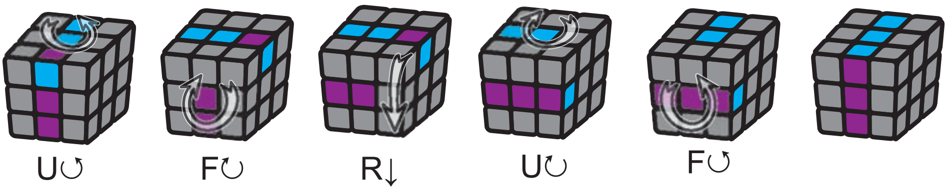


Slice: A layer that is not a face (see above). Odd-sized cubies will have a **middle slice**. We tend to number our slices based on a parallel face layer (e.g., “U2”), and use a lowercase “s” to refer to all the slices parallel to that direction when moved as a group (e.g., “Fs”).

Move: Rotating a single layer (or, for the larger cubes, a group of connected layers). Occasionally we’ll use the word in its generic sense (e.g., “our next goal is to move the DR edge to the UF position”).

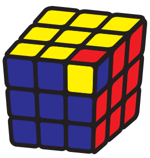
Turn: Some solvers use this as a special term, to mean a move that is a simple 90˚ rotation between two layers (hence, a 90˚ **slice** move is two “turns,” and a 180˚ rotation is two “turns”). We don’t use this as a special term, but we’ll sometimes use it for moves (”turn the U layer clockwise”) and we’ll occasionally use it to refer to reorienting the entire cube, e.g., “turn the cube upside-down so that the UFR corner is now the DFL corner.”

Sequence: A series of moves that changes the cube in a predictable way. Most methods of solving the cube involve looking for the positions of certain cubies on the cube, then executing sequences to move them into the correct position (and/or orientation).



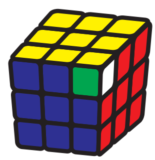
Sequence K_1 from section 3.6, found later in this book

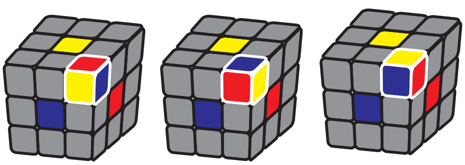
Sequences often focus on changing one section of the cube while ignoring other sections of the cube (which the sequence might mess up); when we depict a sequence, we'll portray the sections to be ignored in gray, as in the diagram above.


The UFR cubie is positioned correctly but not oriented correctly

Positioned Correctly: When a cubie is in the correct position. For example, if the UFR corner cubie (the cubie with the U, F, and R colors on it) is in the UFR position (the corner cubie position common to the U, F, and R layers, then it is positioned correctly. A cubie that is positioned correctly may not yet be **oriented correctly**, though.

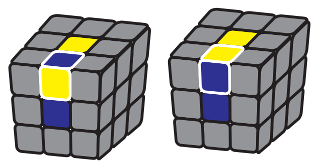
Oriented Correctly: When a cubie has been spun or flipped such that its colors match the face colors. (Usually this term only applies to cubies that are positioned correctly, as it is a bit ambiguous otherwise, but it might occasionaly be used to refer to one face color matching, as in the diagram at right). The Cube is solved when all cubies are positioned correctly *and* oriented correctly.


The UFR cubie is oriented correctly but not positioned correctly


three different ways to spin the UFR cubie

Twist: To re-orient a corner cubie such that it is in the same position but has its colors pointing in different directions.

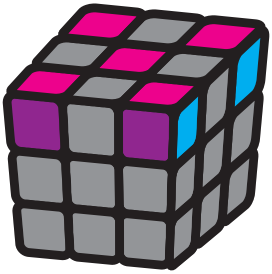
Flip: To re-orient a center edge cubie such that it is in the same position but has its colors swapped.


two ways to flip the FR cubie

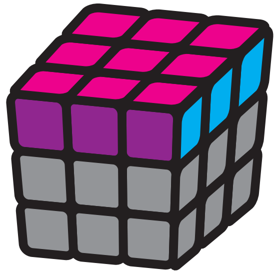
How to Solve the 3×3×3 Cube

Ma~~ke sure you have read the Introduction on the previous four pages before starting! Our method of approach will be as follows. It may be helpful to read each step carefully before starting it for real~~.

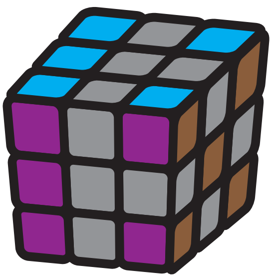
3.1 Solve the U Layer Corners



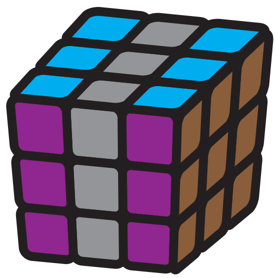
3.2 Solve the U Layer Edges



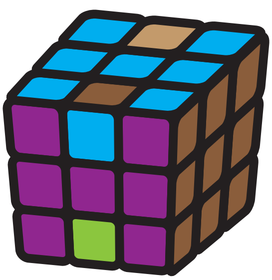
3.3 Solve the Remaining Corners



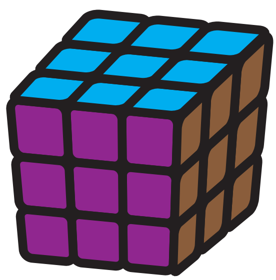
3.4 Solve the R Layer Edges



3.5 Position the Remaining Edges

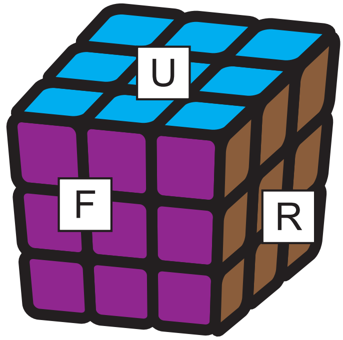


3.6 Orient the Remaining Edges



Advanced Notes: Our method is great for learning and easy to memorize, but it's not so great if you want to break speed records. Speed-cubing systems require a *lot* of memorization and are optimized for few moves on few faces, which has the advantage that fewer moves are needed. However, generalization to larger cubes becomes difficult. We hope that our method will help you understand more *why* than *how*, and help you remember how to solve the cube for years to come. If you're interested in moving into the realm of speedcubing, by all means check out the many web sites on the other methods (the two main families are Jessica Fridrich's layer-by-layer method and Lars Petrus' two-adjacent-faces last method) after you read ours.

3.0 On Notation

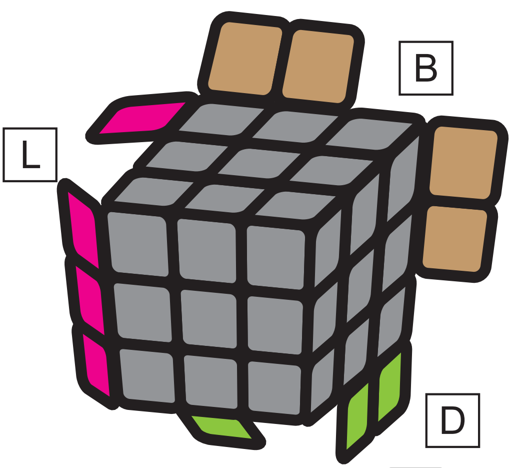


Most solvers tend to hold the cube in the left hand while turning faces with the right hand; ~~accordingly, our solutions are biased towards moves of the~~ Up (U), Front (F), and Right (R) layers. The standard diagram (depicted to the right) ~~according also is optimized for displaying~~ the U, F, and R faces. ~~(If you are a left-handed solver, you might find it convenient to turn the whole cube a bit to the left, so that the F face is to your left, and the R face is facing you.)~~

Since actual cubes come in a wide variety of colors, ~~we won't be so audacious as to assume a standard coloring; hence~~, we'll be using some non-standard colors in our diagrams:

Cyan Purple Brown
Lime Pink Tan

~~Please substitute these mentally for the colors on your cube as necessary.~~ We'll also use Gray for faces we don't particularly care about at the moment.

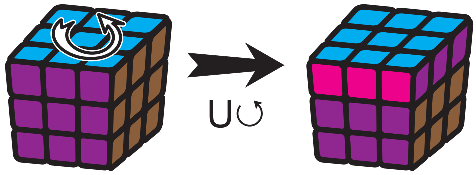


Occasionally we'll need to refer to the other three faces of the cube~~s~~ -- Down (D), Left (L), and Back (B). When depicting them, we'll use the "flap" notation as seen to the left; imagine the hidden faces as if the cube were a partially unfolded box.

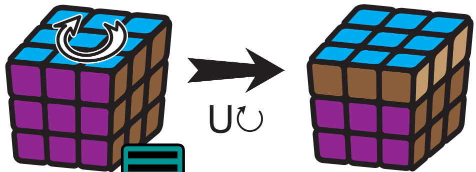
One common problem is mistaking that B stands for "bottom" when it really stands for Back. ~~We'll make this easy on you by never requiring you to turn the Back face, but we'll still need to refer to it occasionally, so be careful.~~

For turns, we'll use the following notation and diagrams:

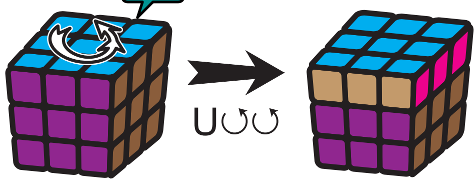
U↺: Turn the Up layer counterclockwise 90°.



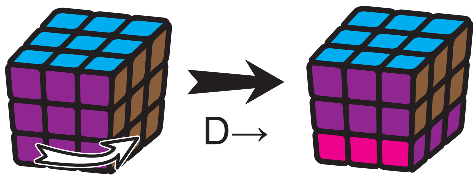
U↻: Turn the Up layer clockwise 90°.



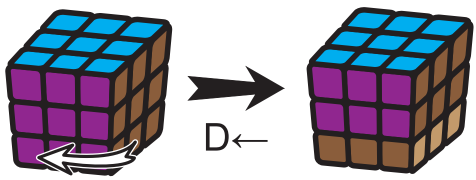
U↺↺: Turn the Up layer 180°. (This is the same as U↻↻.) Note the double-arrowhead in the diagram.



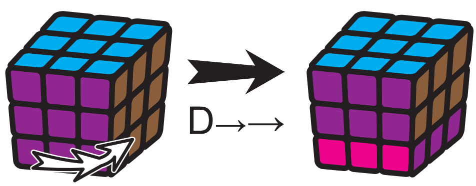
D→: Turn the Down layer to the right 90° (clockwise if you are looking from below).



D←: Turn the Down layer to the left 90° (counterclockwise).

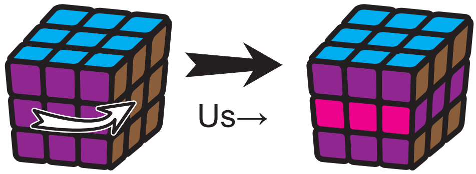


D→→: Turn the Down layer face 180°. (This is the same as D←←.)



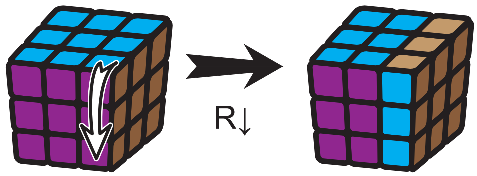
Advanced Notes: Most experts use a different notation for turns (invented by David Singmaster, a contributor to this book), where U means clockwise, U' (or U^{-1}) means counter-clockwise, and U^2 means 180°. We use our notation because you won't need to tilt your cube around to figure out which way is clockwise and we can avoid ambiguity with slice moves, but be aware of this difference when reading about the Cube elsewhere. Singmaster versions of the moves will be supplied at the end of this section.
Also, you might ask, why don't we use the same sort of arrows for U as we do for D? Because a lot of people think of "turn to the right" as *clockwise*, so for us to use U→ to mean *counterclockwise* would be really confusing.

Us→: Move the second layer from the top to the right 90°. This is the same as Ds→, as the second layer from the top is the same as the second layer from the bottom. This in-between layer is often called a “slice”.



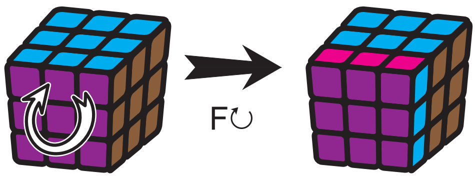
Us← and Us→→ (not shown) follow similarly.

R↓: Turn the Right face downwards (counterclockwise) 90°.



R↑ and R↓↓ (not shown) follow similarly.

F↻: Turn the Front face clockwise 90°.

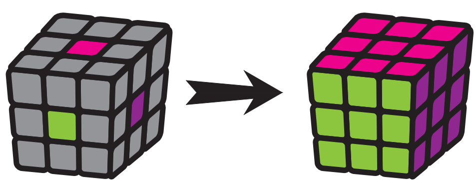


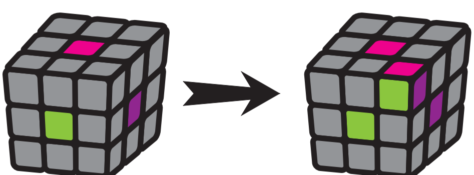
F↺ and F↻↻ (not shown) follow similarly.

We’ll also use Rs and Fs for the two slices in the other dimensions. ~~(A special section regarding slices will show up when we need to use them, in section 3.2).~~

3.1 Solve the U Corners

The 3×3×3 has center face cubies that will determine what the color arrangement in the solved cube should look like:





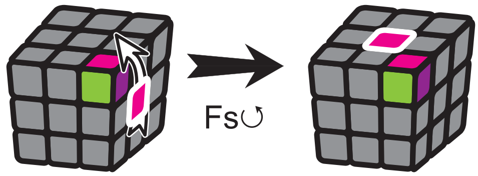
But let’s start out with the modest goal of just getting one corner cubie to match the center cubies next to it. ~~Try this on your own first; this is a good exercise to get familiar with the peculiarities of the cube.~~

~~Go on, try it! We’ll still be here when you get back.~~

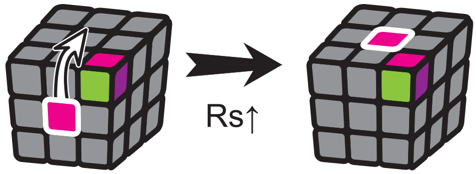
Here's a simple way to do it that ~~probably hasn't occurred to you:~~ *move the center cubies to match the corner*, instead of the other way around.

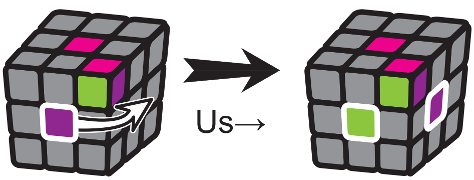
Pick up your cube, and look at the UFR corner cubie. Look at the color that is on the U side of that cubie (in our example, ~~let's say~~ it's pink). Find the center cubie that has that color.

If that center cubie is on the R, D, or L face, simply repeat Fs↻ to bring it to the U face.



If it is on the F, D, or B face, simply repeat Rs↑. ~~(Yes, both ways work for the D face. Can you figure out why?)~~

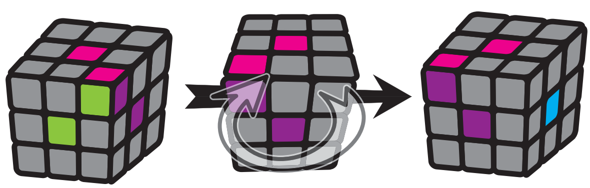




Once the U face cubie is in position, repeat Us→ until the F and R face cubies match the UFR corner cubie. ~~(Unless someone has messed with the pieces or stickers, in which case we can't help you.)~~

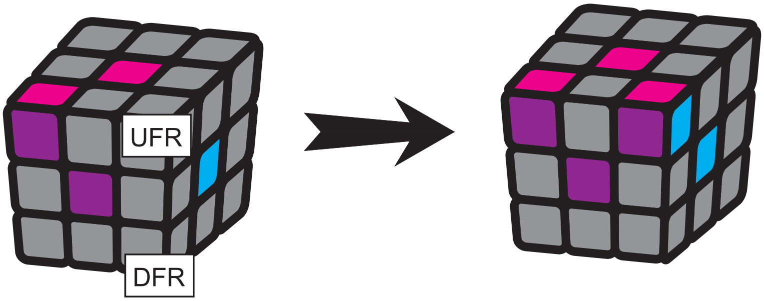
Congratulations! You've now positioned (and oriented) the UFR cubie correctly. ~~The same technique isn't going to work for the other U layer corner cubies, however.~~

Turn the entire Cube (clockwise from the top) so that the UFR cubie is now the UFL cubie.

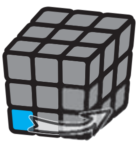


Our next goal is to find the cubie that should go in the (new) UFR position and solve that. It should be another corner cubie with the U and F color ("pink" and "purple" in our example), and the third color should match our new R color. Once you have found the cubie, see if you can get it there yourself without disturbing the UFL cubie or moving the face cubies. If you're stuck (or even if you're not), go on to the next section.

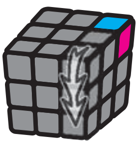
3.1.1. Solving the UFR Corner Cubie



Assuming the UFR-colored cubie isn't in the correct place already, your task should be to put it in the DFR position. You should be able to do this without disturbing the UFL cubie, by simply turning the B (back) face, R (right) face, and D (down) face as needed. Here's how:



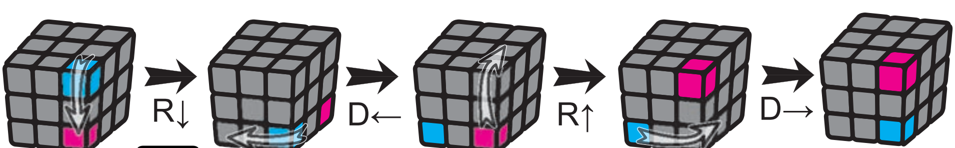
If the cubie is anywhere on the Down layer, simply doing D→ (turning the Down layer to the right) repeatedly will bring the cubie to the DFR position eventually.



If the cubie is in the UBR position, then R↓↓ will bring it to the DFR position.

And if the cubie is hiding in the UBL position, then you can turn the B (back) 180˚ to bring it do the Down layer, then turn the Down layer to bring it to the DFR position.

Now that you have the cubie in the DFR position, here's a simple sequence that swaps the UFR cubie with the DFR cubie. (For this sequence, we've colored those two cubies one consistent color so you can see how they swap.)

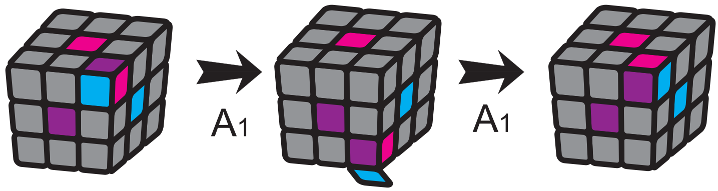


We'll call this sequence A_1.

$$A_1 = R\downarrow D\leftarrow R\uparrow D\rightarrow$$

Using A_1, you should ~~easily~~ be able to get the cubie from DFR to UFR (its correct position), but it is possible that the cubie is in the wrong orientation. What can we do about that?

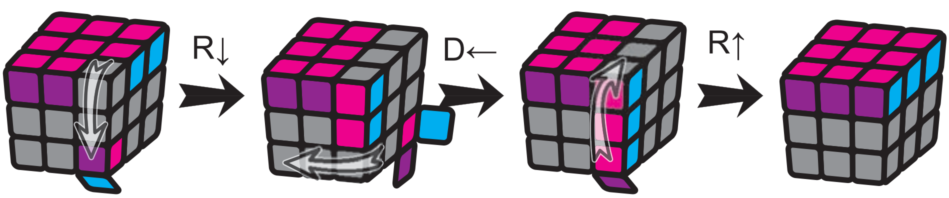
The answer is: by ~~simply~~ repeating A_1, we have a move sequence that twists the UFR cubie counterclockwise:



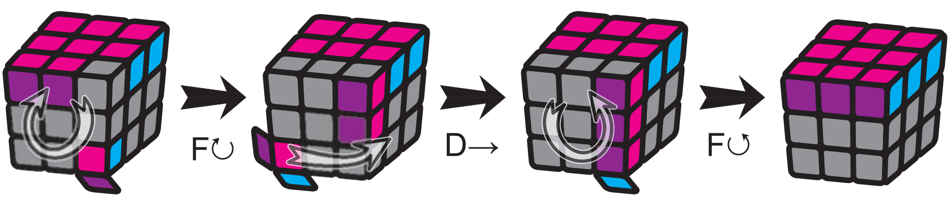
We'll call this sequence A_2:

$$A_2 = A_1\,A_1 = R\!\downarrow\ D\!\leftarrow\ R\!\uparrow\ D\!\rightarrow\ R\!\downarrow\ D\!\leftarrow\ R\!\uparrow\ D\!\rightarrow$$

Advanced Notes: You might have noticed that if all we care about is getting the DFR cubie "up" to the UFR position, the fourth move in A_1 seems unnecessary; why turn the Down face when we don't care about it anymore? Advanced solvers, in fact, don't bother with that fourth move, and will often do just R↓ D← R↑ instead if they don't care about the D face:

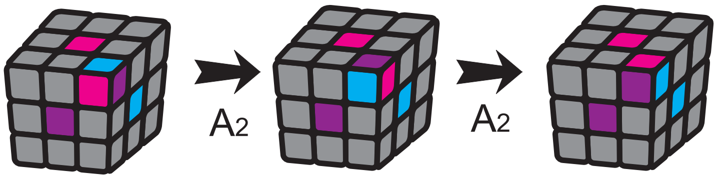


Furthermore, based on how the cubie is oriented, a similar sequence might be used instead:



This is actually the same sequence, but "mirrored" in a way that swaps F with R. This "mirroring" technique ends up being quite useful for sequences that relate to moving corner cubies.

If the UFR cubie is still in the wrong orientation after A_2, another application of A_2 should fix it:



UFR cubie should now be positioned and oriented correctly and matching the UFL cubie.

Advanced Notes: You might have noticed it strange that it takes 8 moves (A_2) to twist the UFR cubie counterclockwise, but 16 moves (A_2A_2) to twist it clockwise. By doing A_2 *backwards*, however, we can do the clockwise twist in 8 moves:

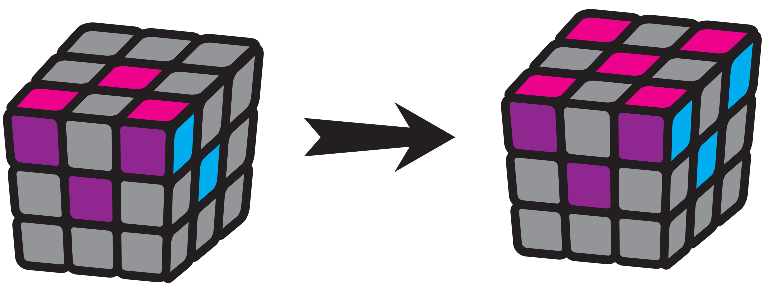
anti-A_2 = D← R↓ D→ R↑ D← R↓ D→ R↑

Also, since we don't really care about the other pieces in the D layer, we can use the F/R mirror trick mentioned in the last box of notes, and combine both sequences to do the twist in just 6 moves:

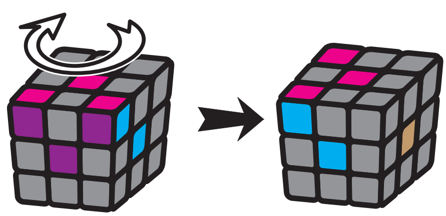
alternate A_2 = F↻ D← F↺ R↓ D← R↑ (twists UFR counterclockwise)

anti-alternate A_2 = R↓ D→ R↑ F↻ D→ F↺ (twists UFR clockwise)

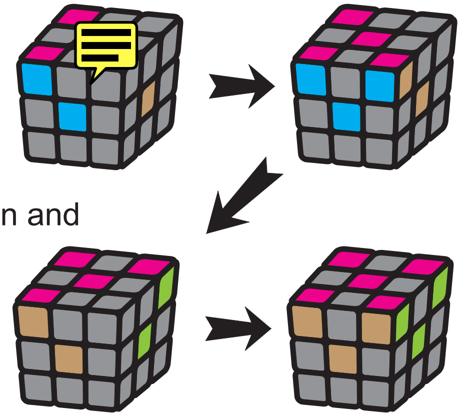
3.1.2. Solving the Other U corner cubies



Once you have the UFL and UFR cubies correct, the next thing to do is turn the whole cube clockwise ~~around the U face,~~ so that ~~what were~~ the two cubies you solved become the UBL and UFL cubies.



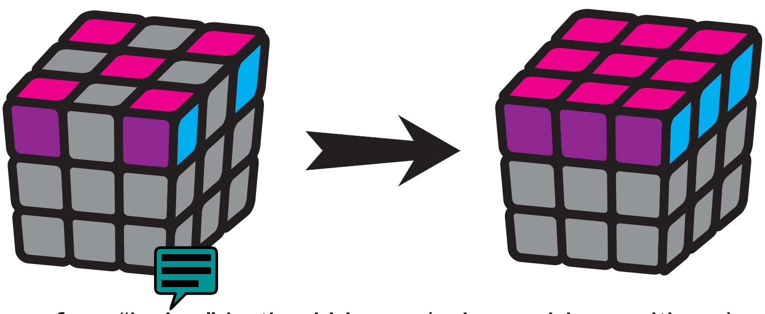
This now puts a new color on the F and R faces, which means we have a new UFR position to fill. We can re-use the moves in the previous section to solve the new UFR cubie (put it in the DFR position and repeat A_1/A_2 as necessary).



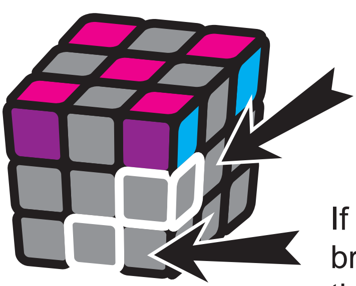
By repeating the process again (turning the whole cube and solving the new UFR corner), we can then complete the last corner cubie in the U face. Now all the corners in the U face are solved!

Reminder:
A_1 = R↓ D← R↑ D→

3.2 Solve the U Layer Edges



There are four ~~“holes”~~ in the U layer ~~(edge cubie positions)~~, and our next goal is to fill them with the appropriate edge cubies. Look around your cube and see if you can find an edge cubie with the U color (in our picture, pink) that isn't already on the U layer. Turn either the Us slice or the D ~~(Down)~~ layer so that the cubie is in one of these two positions:



If the cubie is in the Us ~~(middle)~~ layer, bring it to the FR position by turning the Us slice. (Don't worry about the center cubies for now.)

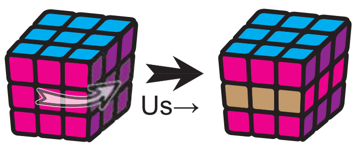
If the cubie is in the D ~~(down)~~ layer, bring it to the FD position by turning the D layer.

A Digression: The Power of the Slice

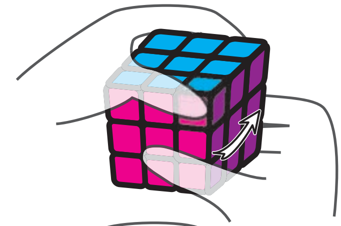
The slice moves get scant respect among the Cube community. The reasons for this are twofold; the speed-solvers don't like slice moves because they can't be executed with one finger quickly, and the theoreticians don't like them because they like keeping the center face cubies stationary so that notation is simpler.

But the slice move is excellent for new solvers trying to understand the Cube, for one simple reason -- *the slice moves don't disturb the corner cubies*. Since solving the 3×3×3 is simply solving the 8 corners and 12 edges, using moves that only affect edges allow for beginners to more easily follow what's going on in these sequences and understand how they work. This becomes even more invaluable when you move up to the bigger Cubes.

However, having an easy-to-visualize move isn't much use if you can't execute the move cleanly and simply. So, here's a quick guide on how to make some slice moves:

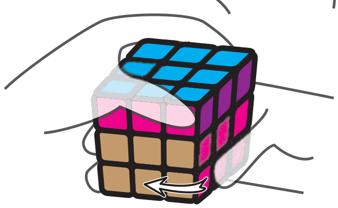


To do Us→, hold the U layer steady with your left hand, while your right hand grips the Us and D ~~(Down)~~ layers. Your right thumb should be between both layers, while your right index and middle fingers are on the Us and D layers, respectively.

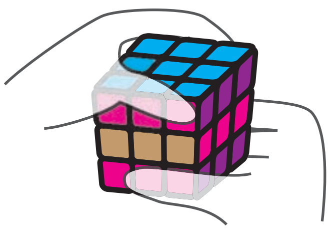


Turn both the Us and D layers to the right a quarter-turn.

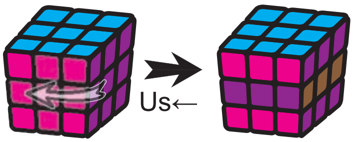
Without releasing the cube, shift your right thumb down a bit, and release pressure on your right index finger.



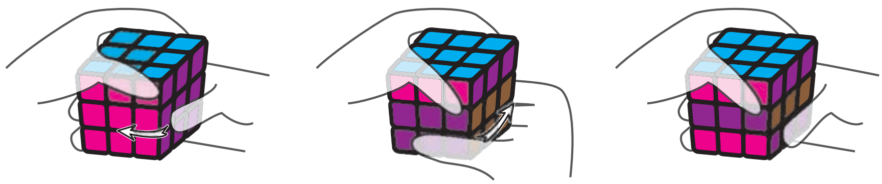
Now turn the D layer back left, using just your right thumb and middle fingers. (You can slide the ring finger of your *left* hand down a bit to keep the Us layer from turning.)

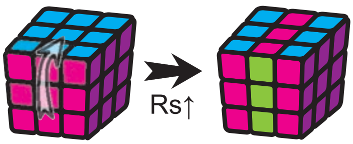


This two-turn combination should perform Us→ suitably. Practice this a few times to make sure you have the movement down pat as one fluid motion.

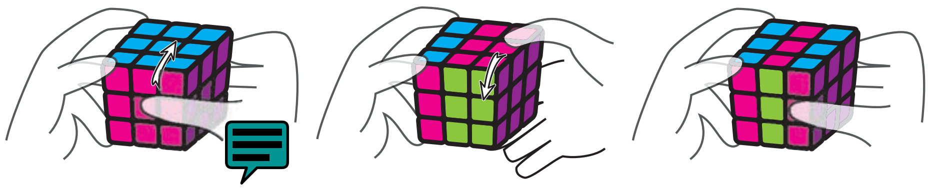


Us← is similar, but this time start your right hand in the rear position, and slide your left thumb down a bit to hold the Us layer in place when you turn the D layer back.

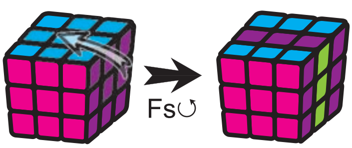




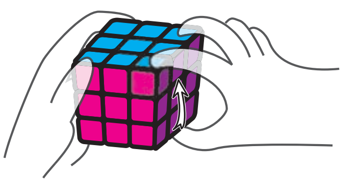
Rs↑ can be done by holding the L layer with your left hand, while the right hand turns both the Rs and R layer up, then the R layer down after retracting the right-hand grip a bit.



~~Figuring out Rs↓ is left as an exercise to the reader.~~

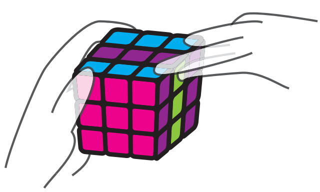


Fs↻ is a bit trickier to deal with. You can either use a similar method to the Us and Rs slice moves, where the left hand holds one face steady while the right hand does a back-and-forth turn, or you can use this alternate method:



Grip the L layer with the left hand, making sure not to touch the Fs slice ~~(easy)~~.

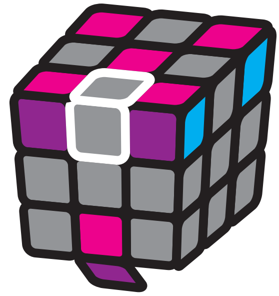
Have your right hand in a “claw” position, where your thumb is on the D (down) side of the DR cubie, and your index and ring fingers are on the U side of the UFR and BFR corner cubies.



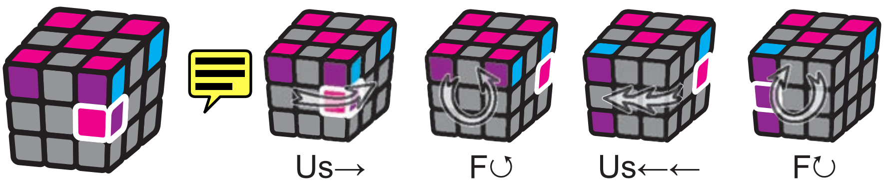
With one move, squeeze the “claw” by pulling your right thumb up. This should make Fs↻ in just one move!

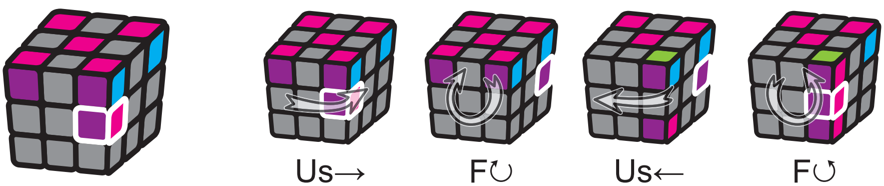
To do Fs↺, simply hold the “claw” upside-down, with your thumb on UR.

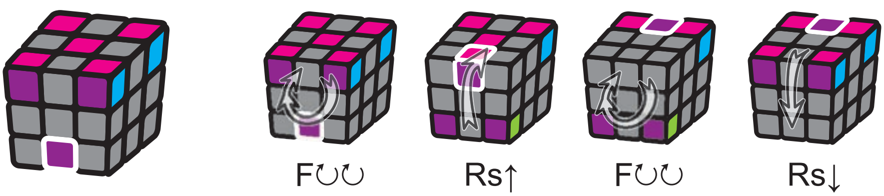
Inspect the edge cubie and look at what the other color is on the edge cubie. That should tell you which ~~“hole”~~ in the U layer it should go into. Turn the U layer so that the ~~“hole”~~ is in the UF position. In the example at right, our edge cubie (in the FD position) has a ~~lime~~ color, so we position the ~~lime~~ “hole” at the UF position (outlined in white).

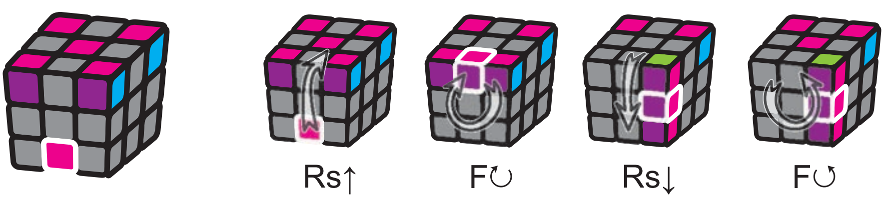


Now you should be able to use one of these four 4-move sequences, based on the position and orientation of your edge cubie:

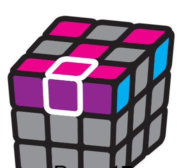






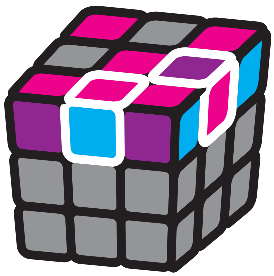


Find the diagram on the left that matches the position of the UF edge cubie. Then follow the four steps to the right of that diagram. The UF edge cubie should now be positioned correctly! (Check to make sure your corners are still safe.)



Advanced Notes: Advanced solvers will note that these aren't the fastest sequences that work. For example, F↻ Us→ F↺ is clearly better than Rs↑ F↻ Rs↓ F↺. We chose these because the parallelism between these makes them a bit easier to remember. As you get better at visualizing the cube, try to come up with moves that can move an edge cubie from anywhere to the UF position, without necessarily bringing it to the FR or DR position first.

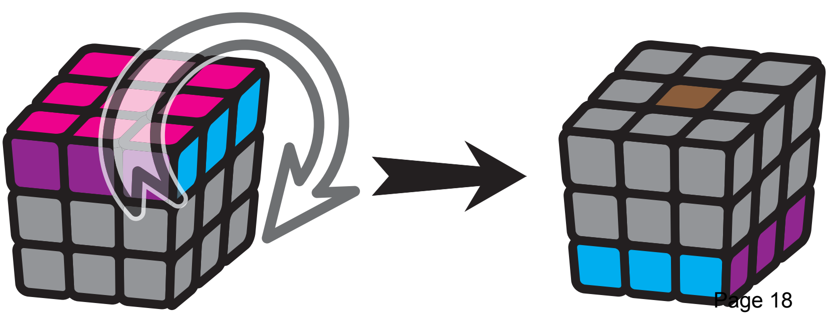
~~It is possible that you might have U edge cubies that are already in the U layer, but in the wrong position or wrong orientation (see example at right). What can we do about this? Well, since the previous sequences move an edge cubie *into* the UF position, it stands to reason they all move whatever was in the UF position *out* to a different position. So, simply use any of those sequences to move an edge cubie out, and use the appropriate sequence to move it back in, correctly this time. (Or, use the G sequences from section 3.4., seen later, for another way to do it.)~~



~~By repeating this process, you should eventually be able to bring all the U edges up to the U layer and in the correct place, solving the entire U layer! Now it's time to move on to solving the opposite layer.~~

3.3 Solve the Remaining Corners

First, turn the cube upside-down, so that the layer you have solved is now the D layer:

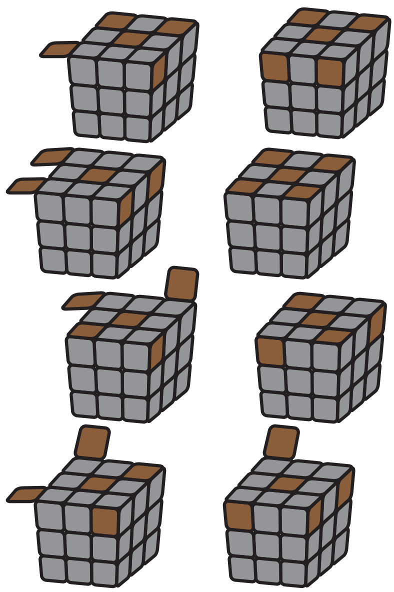


The four remaining unsolved corners had better be in the U layer (after all, there's nowhere else for them to go), but they probably are not in their individually-correct positions. Even if they are *positioned* correctly, they probably aren't *oriented* correctly yet. We will position them correctly first, and then fix their orientation later.

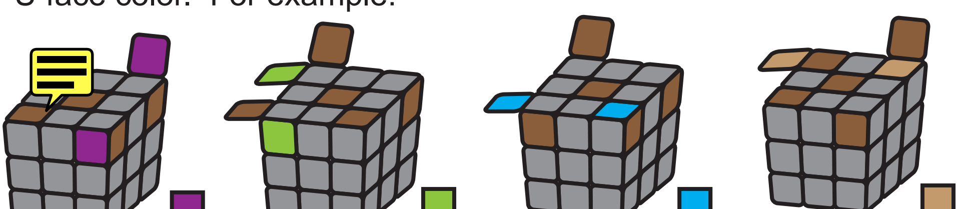
3.3.1 Position the Remaining Corners Correctly

~~First, let's determine which color needs to end up on top. That's easy; it's the color of the U center (face) cubie (brown in our example). Look at the four U corner cubies and confirm that each one has that color somewhere (see inside the box below for examples). Ignore the edge cubies on the U layer for now; we'll fix them in section 3.4.~~

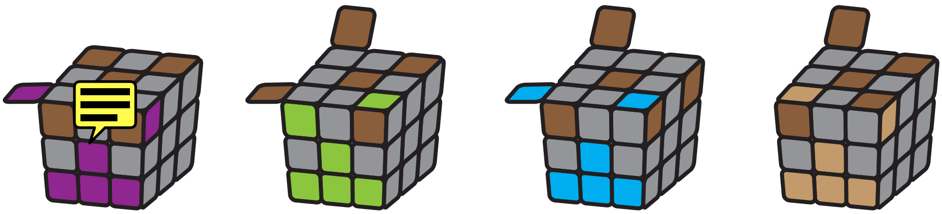
Advanced Notes: It turns out that, barring rotations of the U layer, these eight coloring arrangements are the only possible ways the color panels of the U face corner cubies can be positioned. One consequence of this is that, if you see a configuration that is not one of these eight (such as having three corner faces on top with the U color), then some joker has probably disassembled and reassembled your cube in an unsolvable position!



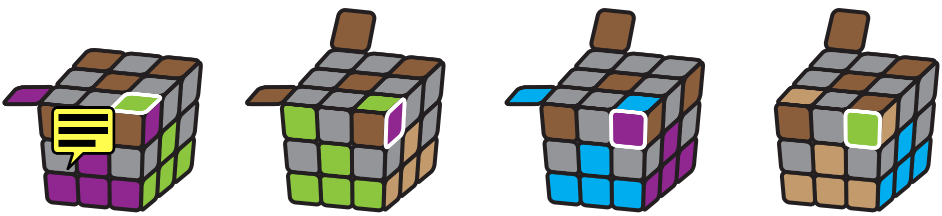
~~Now, look at the *other* colors on the four corner cubies. Find a pair of corner cubies on the same side (that is, not diagonally apart from each other) that have a color in common besides the U face color. For example:~~



~~Turn the U layer (or the whole cube) so that those two corner cubies are up front (in the UFL and UFR positions), then turn the Us slice layer so that the center face cubie matches the color, and finally turn the D layer so that the DF position cubies also match the color. Using the examples from above:~~

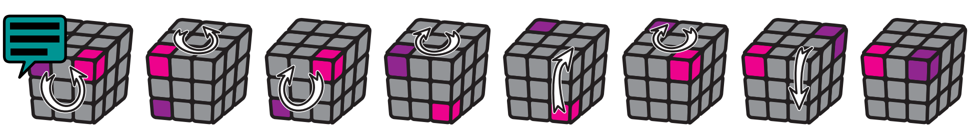


Now you have to find out whether the UFL and UFR cubies are in the correct position (regardless of orientation), or if they have to be swapped. Look at the three colors on the UFR cubie. If you've done everything right up to now, one will be the U color and one will be the F color. What about the third? Is it the R color (you can determine the R color by looking at the R center cubie or the RD cubies)? If it is, then the UFR is in the correct position (though it may have to be spun to a new orientation); if the third color on UFR does not match the right bottom row, then we will have to swap UFR and URL.



Matches R; no swap needed | Doesn't match R; swap needed | Matches R; no swap needed | Doesn't match R; swap needed

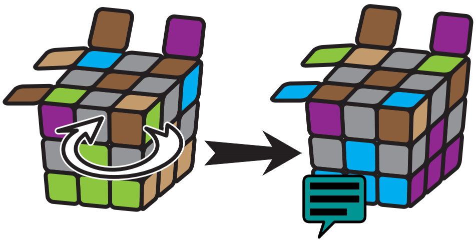
To swap the UFL and UFR cubies, use this sequence (in the diagram, we've colored the UFL and UFR cubies with a uniform color so you can see how they move):



F↺ U↺ F↻ U↻ R↑ U↻ R↓

Call this sequence C: $\boxed{C = F↺\ U↺\ F↻\ U↻\ R↑\ U↻\ R↓}$

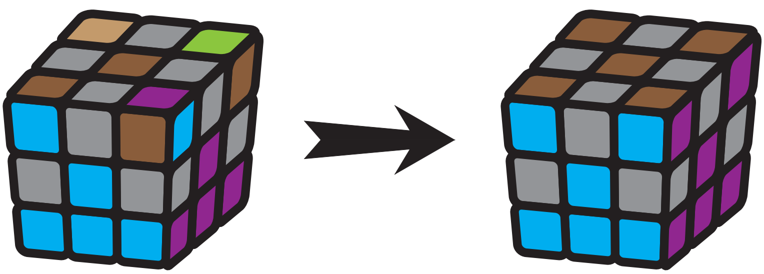
After placing the UFL and UFR cubies correctly, we'll need to look at the other two cubies on the U layer (the UBL and UBR cubies), since they might also need to be swapped. To check this, simply turn the cube 180 degrees, keeping the U face up:



Now the UBL and UBR cubies have become the UFR and UFL cubies. Check to see if they need to be swapped. If so, use sequence C again to do so.

Now the U layer corners cubies are placed correctly but (unless you are very lucky) are not oriented correctly.

3.3.2 Orient the Remaining Corners Correctly

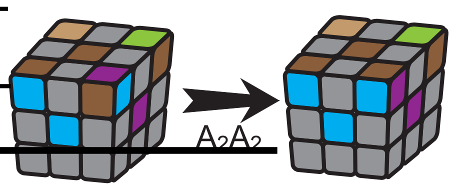


This can be done in a straightforward way based on our old friend A_2 from 3.1.2:

$$A_2 = R\downarrow\ D\leftarrow\ R\uparrow\ D\rightarrow\ R\downarrow\ D\leftarrow\ R\uparrow\ D\rightarrow$$

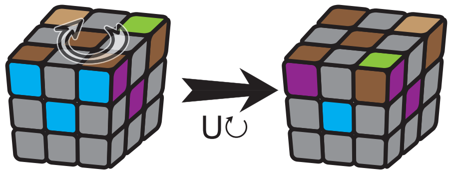
Recall that the main feature of this sequence is that it twists the UFR corner counter-clockwise. ~~It leaves the rest of the U layer untouched, but *does mess up the D layer* -- in a predictable way such that repeated uses of A_2 will restore it. Keep careful track of the U and F faces as you go through this section, because you won't have your nice clean D layer to rely on!~~

~~If the UFR corner needs to be spun (you can tell by whether or not the U color is on top), apply A_2 until it is oriented correctly. (Don't omit the D turn in A_2 or you'll be sorry!)~~

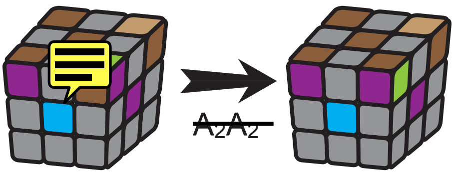


~~This will certainly destroy the cubies in the D (down) layer. Do not panic! If you are careful, it will be restored at the end.~~

After orie[illegible]g the UFR cubie correctly., turn *only* the U layer clockwise (U↻) so that there is a new cubie in the UFR position. (Don't turn the Us layer or the whole cube!)



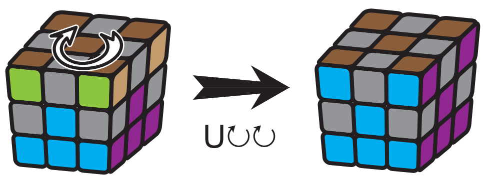
If the new UFR cubie is already oriented correctly (the U color will be on top), just do U↻ again until there is a cubie in the UFR position that needs to be oriented correctly.



~~Now, apply A_2 more until the (new) UFR cubie is oriented correctly (with the U color on top).~~

Continue in this fashion, turning *only* the U layer to bring a new cubie to the UFR position, using A_2 to spin it, and so on. Eventually all the U-layer corner cubies will be oriented correctly; and, as if by magic, the D-layer cubies will be restored as well.

Finally, turn the U layer as necessary to match up all the corne[illegible]ll the corners are now solved~~; now only the edges are le~~ft!



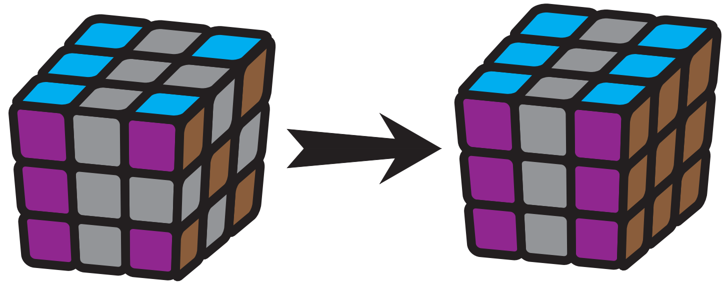
Advanced Notes: This method of re-orienting the top corners is simple but can take up to six applications of A_2 and four turns of the U layer, a total of 52 face turns. However, it turns out that A_2A_2 and anti-A_2 have exactly the same effect, so by using anti-A_2 as necessary we can reduce the maximum number of turns to 28. By using the alternate A moves mentioned in a previous note, the number of moves can be reduced even further! Speed-cubers go one step further and memorize moves that twist (and flip) multple cubies at once. One old standby is this sequence, called Sune™ by Lars Petrus when he rediscovered it in 1980:

R↑ U↻ R↓ U↻ R↑ U↻↻ R↓ U↻↻

This sequence twists three cubies in the top layer clockwise while leaving the D layer untouched. The versatility of this sequence is that all of the 8 orientations can be solved by at most two applications of Sune or anti-Sune; and that Sune can be executed very rapidly as only two faces (R and U) are ever turned.

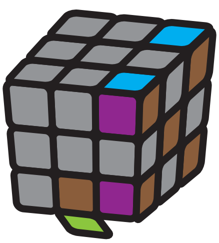
Before you begin the next section, turn the entire cube so that the solved ~~(Down)~~ layer is now the L ~~(Left)~~ layer.

3.4 Solve the R Layer Edges



At this juncture, there are only eight unsolved edge cubies left; four on the R layer, and four on the Rs slice. Our next task is to find which four edge cubies that should be on the R layer, and then move them to their proper place *including* correct orientation. (We'll deal with the Rs slice in the next section, so don't worry about messing it up now.)

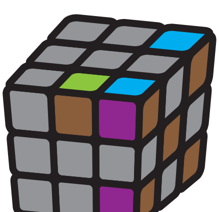
~~Identifying which cubies need to go to the R layer should be easy: if the edge cubie has the R color, it belongs on the R layer. We'll start by solving one of them.~~



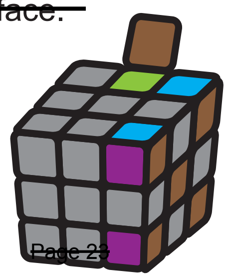
~~See if you can find such a cubie among the four cubies on the Rs slice. In the diagram at left, we've found one at the FD position.~~

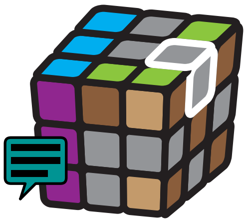
~~(It is possible, if you're unlucky, that all the R layer edge cubies are already on the R layer, so you won't be able to find one on the Rs slice. If that is the case, fake it by imagining that you've found one, and continue following with us.)~~

~~The edge cubie you've found should have the R color on one side and another color panel on the other side, which should match one of the other four unsolved faces. (In our diagram above, the other color is lime.) Turn the Rs slice until this panel is on the U face.~~

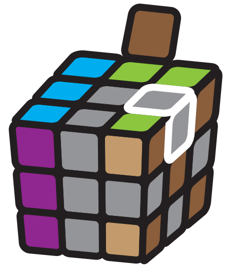


~~Depending on how that edge cubie was oriented, the cubie will either end up in the UF position (seen at left) or the BF position (seen at right).~~



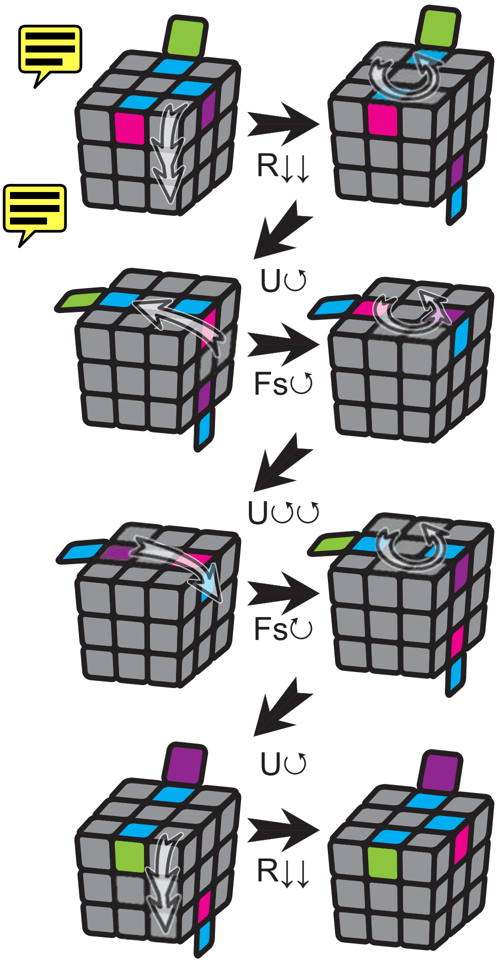


~~Next, turn the R face until the two corners on top match that color -- in other words, the "destination" for~~ our edge cubie ~~is now the UR edge.~~

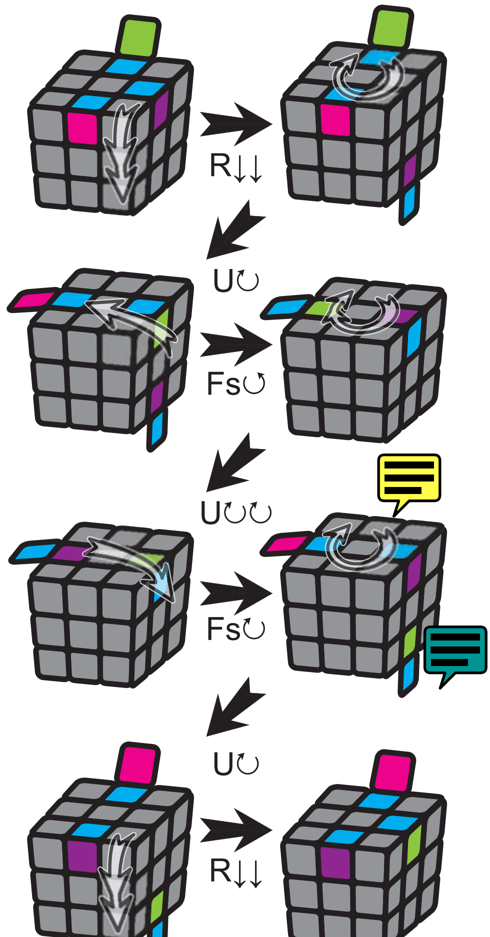


Now we're ready to apply the next sequence, which moves, UF, UR, and UB in a cycle. This comes in two varieties, which move the three edges counterclockwi ~~clockwise, repsectively~~:

G_1 = R↓↓ U↻ Fs↻ U↻↻ Fs↺ U↻ R↓↓



G_2 = R↓↓ U↺ Fs↻ U↺↺ Fs↺ U↺ R↓↓

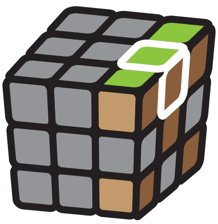


Some solvers may find G_1 and G_2 inconvenient because of the Fs moves. Here are some alternate versions that do the same thing, but “fake” the slice by turning the F and B faces:

alt-G_1 = R↓↓ U↺ F↻ B↻ R↓↓ F↺ B↺ U↺ R↓↓

alt-G_2 = R↓↓ U↻ F↻ B↻ R↓↓ F↺ B↺ U↻ R↓↓

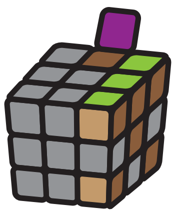
(B↻ here is “clockwise when looking at the front of the cube,” not “clockwise when looking at the back of the cube.” You can see why we avoid using B in sequences.)



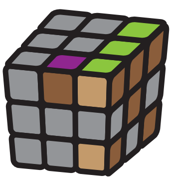
By using G_1 and G_2, you have now solved one of the four edges on the R layer. Well done!

(Or, if we told you to “fake it” earlier, you have now ejected an R layer cube out into the Rs slice, so now you can do the process again, for real this time.)

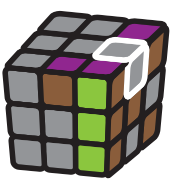
The next thing to do is to repeat the process:



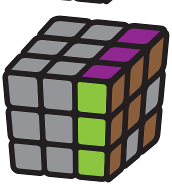
Look for another edge cubie on the Rs layer that is supposed to go to the R layer (it will have the R color somewhere).



Turn Rs until that edge cubie is on the U layer but the R color isn’t on the U face (here, we have purple on the U face and brown on the F face).



Turn R until the destination for our edge cubie is on the U layer.

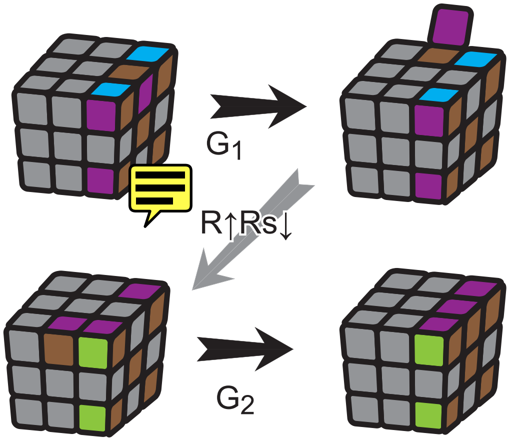


Use G_1 or G_2 as appropriate, and one more edge cubie is solved.

Do this for all four edges, and the R layer should be solved!

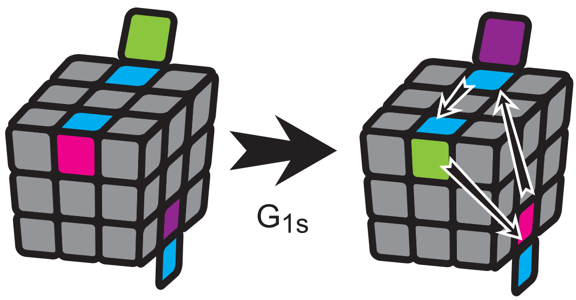
Again, just like as in Step 3.2, you might find yourself with some edges already in the R face, but placed incorrectly. As before, you'll have to use a G move to get it out of its spot, then put it in its spot correctly.

Your R edges are now all solved!



Advanced Notes: After doing a lot of G sequences, you may notice that it sure seems like you're turning the R face back-and-forth a lot. And you would be correct. The only reason the R↓↓ moves are at the beginning of G is so that all the affected edges are on the U layer, making it easier to visualize. Without those moves, it is the RD edge cubie instead of the RU cubie that gets involved. If you can deal with that and not get confused; you'll save yourself a lot of unnecessary R moves by using the simplified G sequences, as shown to the right.

G_{1s} = U↺ Fs↺ U↺↺ Fs↻ U↺
G_{2s} = U↻ Fs↺ U↻↻ Fs↻ U↻



In fact, if you look carefully, you'll see a similarity between these simplified G sequences and the H sequence in the next section; they're the same sequence, just with an extra U turn and using Fs instead of Rs.

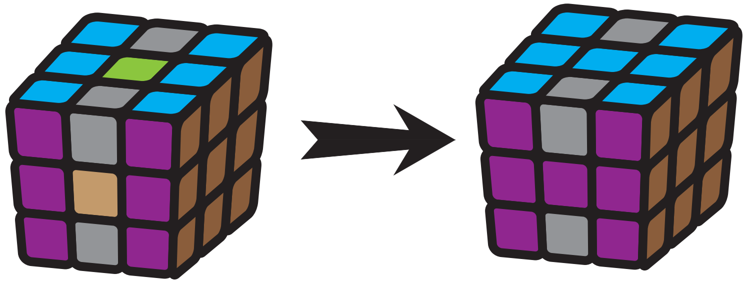
Speed-cubers use this sequence instead:

G_1 = F↻↻ U↺ F↺ U↺ F↻ U↻ F↻ U↻ F↻ U↺ F↻

More moves, but only two faces are turned, making it very easy to do fast. It's very hard to see exactly *how* this sequence accomplishes the same thing as our G_1, though!

3.5 Position the Remaining Edges

First, turn the Rs slice so that the centers match up:

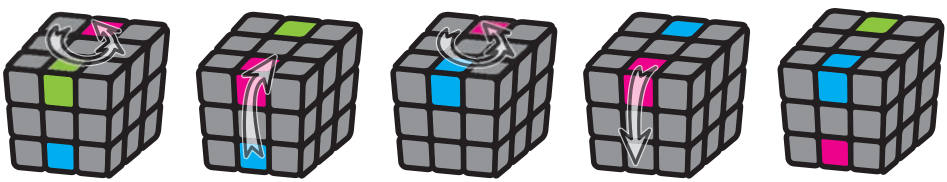


Now, by inspecting the remaining four edge cubies, you should be able to tell which ones are in the correct position, and which ones are not. It's possible for an edge cubie to be in the correct position but "flipped" (oriented the wrong way); count those as correct; we'll deal with the "flipping" in the next section. Depending on how many edge cubies are correct, you'll do different things:

4 If all four of your edge cubies are already in the correct position, great! Move on to the next section.

1 If one of your edge cubies is correct, then the other three need to be permuted. Re-orient your cube so that the correct edge cubie is in the BD (back-and-down) position. If the permutation you need is "upwards" (FD needs to go to FU which needs to go to BU), you can use this sequence:

H = U↻↻ Rs↑ U↻↻ Rs↓



If you need a "downwards" permutation, you have several choices:
(1) You can reorient your cube so that the U and F faces are switched, then do H just once.
(2) You can do H twice.
(3) You can do the inverse of H (Rs↑ U↻↻ Rs↓ U↻↻).

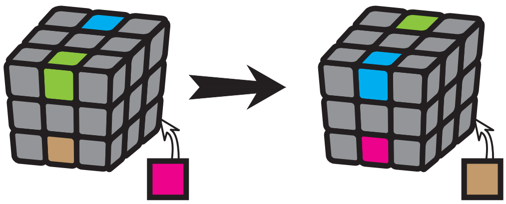
Choose whichever is easiest for you.

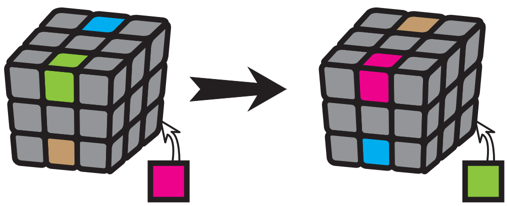
0 — If none of your edges are correct, you'll have to do a bit more work. Repeat sequence H (in the previous section) until one edge is correct, then (as per the previous section) put that edge in the BD position and apply H as necessary.

Advanced Notes: If you don't mind learning some new sequences, here are two that will make the 0 case less tedious:

U↻↻ Rs↑↑ U↻↻ Rs↓↓

Swaps UF with UB and DF with DB.





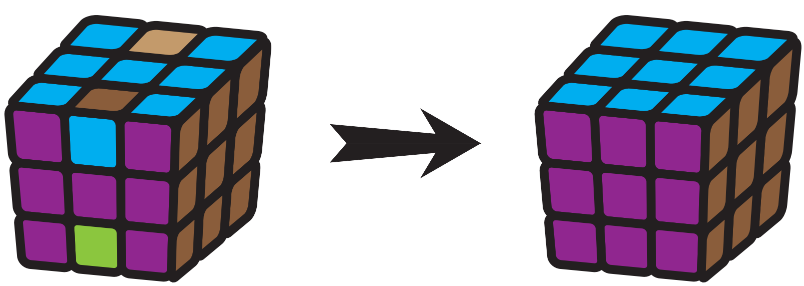
Us→→ Rs↑ Us→→ Rs↑

Swaps UF with DB and DF with UB.

2 — If exactly two of your edges are correct, some practical joker has probably disassembled your cube and put it back together in an impossible-to-solve configuration. Solving such a cube is outside the scope of this section.

3 — If exactly three of your edges are correct, some practical joker has probably swapped the *stickers* of your cube with the stickers of another cube. (Think about it -- how can just *one* item be wrong if there's no other place for it to go?)

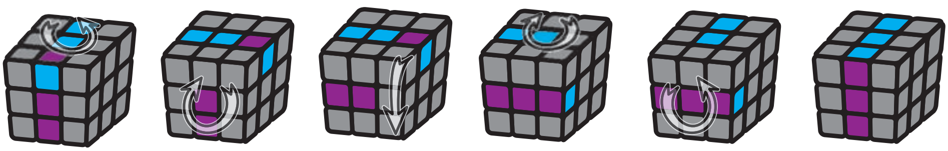
3.6 Orient the Remaining Edges



If your cube isn't solved yet, then either 2 or 4 of the edge cubies are flipped -- an even numbe [illegible] an odd number of edge cubies are flipped, your cube has been tampered with and cannot be solved using only face turns.) ~~Turn the Rs slice (or re-orient the cube) until one of the edges that needs to be flipped is in the UF position.~~

~~The core sequence you need to learn here is a move that flips the UF edge, leaving the rest of the Rs slice intact:~~

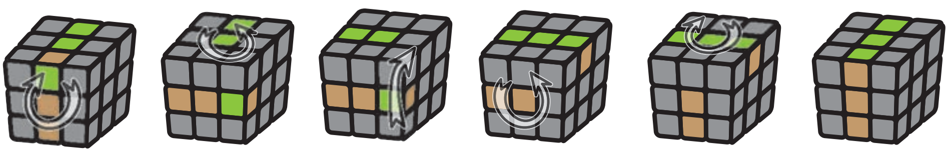
~~K_1 = U↺ F↻ R↓ U↻ F↺~~



~~U↺ F↻ R↓ U↻ F↺~~

~~However, although it leaves the Rs slice intact, it will leave the rest of [illegible] cube in apparent chaos. Do not panic! Simply turn the Rs slice (leaving the R and L layers in place!) so that a different needs-to-be-flipped edge is in the UF position, and then do the inverse of K_1:~~

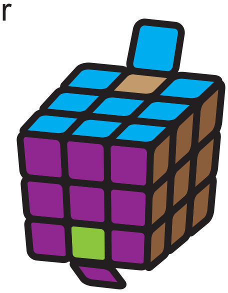
K_2 = anti-K_1 = F↻ U↺ R↑ F↺ U↻

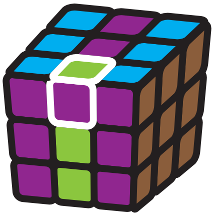


F↻ U↺ R↑ F↺ U↻

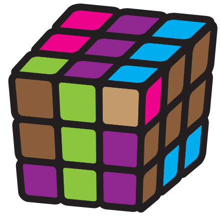
This should flip the other edge while the rest of your cube gets magically restored.

[illegible] case this description leaves you a little squeamish, here's a more specific example. Let's suppose that you need to flip the FD (purple-lime) edge cubie and the UB (cyan-tan) edge cubie, as shown to the right:

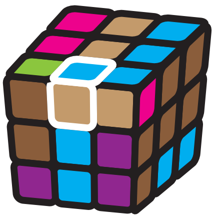




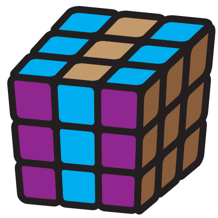
First, do Rs↑ so that the purple-lime edge (one edge that needs to be flipped) is in the UF position, so we can prepare to flip it.



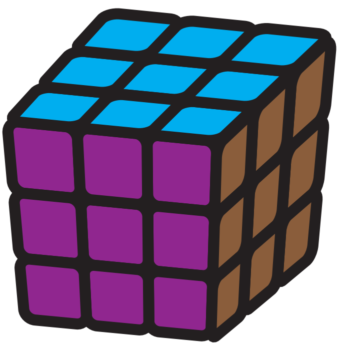
Then, do K_1 (U↺ F↻ R↓ U↻ F↺), which inverts the UF edge and messes up the rest of the cube (but note how nice the Rs slice looks!):



Then, do Rs↓↓ to bring the cyan-tan edge to the the UF position:



Then, do K_2 (F↻ U↺ R↑ F↺ U↻), which not only inverts the cyan-tan UF edge, but also undoes the mess done to the rest of the cube:



A simple Rs↑ (cancelling out the Rs↑ and Rs↓↓ you did previously) and you're done!

Advanced Notes: Flipping two opposite edges on the same face using K sequences takes 14 face-turns, which is actually the fewest number of turns possible. But to flip all four edges on the center slice, K sequences end up requiring 28 turns (24 if you're crafty). The theoretical minimum is only 18. Here's one such sequence; see if you can figure out how it does what it does:

U↺ R↓ U↻ Rs↓ F↻ Rs↓ F↻
Rs↓ F↻ Rs↓ F↻ U↺ R↑ U↻

Congratulations!
~~If you've been following carefully up to now~~, you should have a solved cube -- and you should have a little bit more of an idea of how cube-solving methods work!

Bonus Section: Orienting the Center Face Cubies

Occasionally you'll come across a Cube where one or more of the center face cubies has a picture or words on it, and therefore must be oriented correctly in the solution. We won't give a thorough treatment of how to re-orient all the center faces, but here are two sequences (and their theories) that, when applied correctly, can solve all arrangements of center orientations in less than 7 applications.

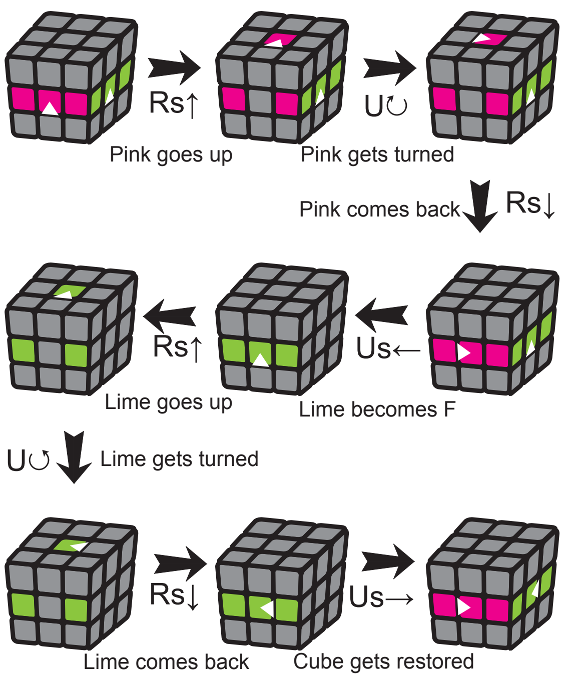
The first sequence rotates the U center face 180˚ by using the R and L layers to spin the cubies around it:

R↓L↑ U↻↻ R↑L↓ U↻ R↓L↑ U↻↻ R↑L↓ U↻

The second sequence turns the F center clockwise and the R center counterclockwise:

Rs↑ U↻ Rs↓ Us← Rs↑ U↺ Rs↓ Us→

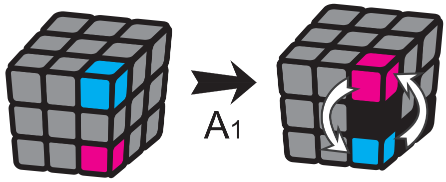
This second one has some principles that are worth diagramming:



Note that you can easily adapt this sequence to handle any two center faces on the Us layer, just by adjusting how much you move Us in steps 4 and 8. You can also change it to turn two center faces 180˚ instead of 90˚ just by using U↺↺ for steps 2 and 6.

Here's a review of all the sequences we've covered. Most of these sequences will mess up sections of the cube; we've colored those sections in black. (Traditional "Singmaster" notation versions of the sequences are given in small print beneath the sequence.)

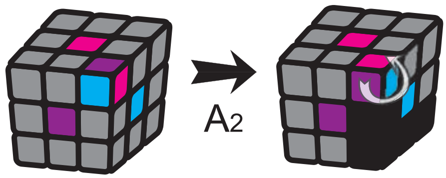
A_1 = R↓ D← R↑ D→
R'D'RD



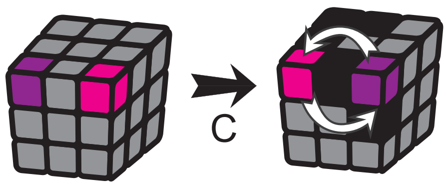
also messes up the DLB and DB cubies

A_2 = R↓ D← R↑ D→
R↓ D← R↑ D→
R'D'RDR'D'RD

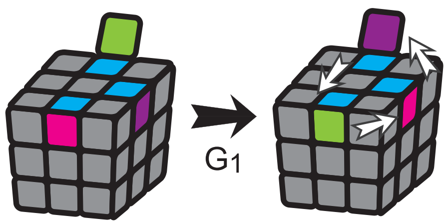
Repeating A_2 three times will restore what it messed up.



C = F↺ U↺ F↻ U↻
R↑ U↻ R↓
F'U'FURUR'

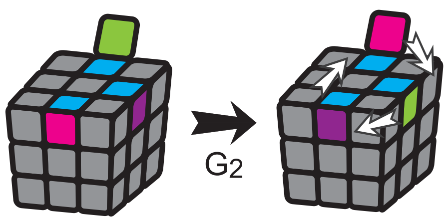


G_1 = R↓↓ U↺ Fs↺
U↺↺ Fs↻ U↺ R↓↓
R2U'FB'R2F'BU'R2

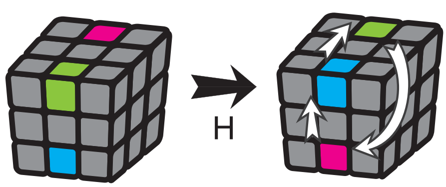


G_2 = R↓↓ U↻ Fs↺
U↻↻ Fs↻ U↻ R↓↓
R2UFB'R2F'BUR2

The G sequences are inverses of each other.



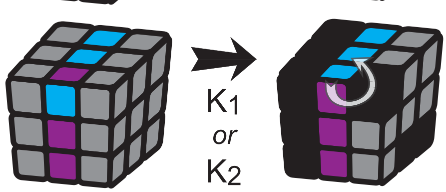
H = U↺↺ Rs↑ U↺↺ Rs↓
U2LR'F2L'R



K_1 = U↺ F↻ R↓ U↻ F↺
U'FR'UF'

K_2 = F↻ U↺ R↑ F↺ U↻
FU'RF'U

The K sequences are inverses of each other.

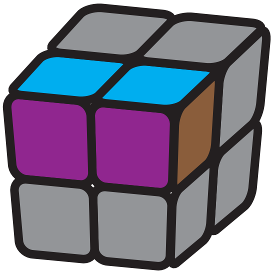


How to Solve the 2×2×2 Cube

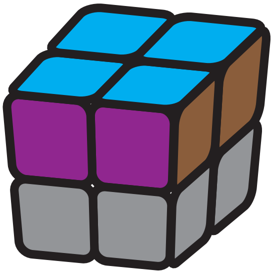
Solving the 2×2×2 cube is extremely similar to solving the corners of the 3×3×3 cube; you might even say it is identical. There is one difference, though; there are no longer any center face cubies to identify the color of each face! Accordingly, although we will mostly use the techniques mentioned in sections 3.1 and 3.3, we'll make special notes on how to identify the face colors.

Our general approach:

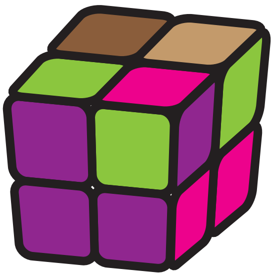
2.1 Solving Two Corners



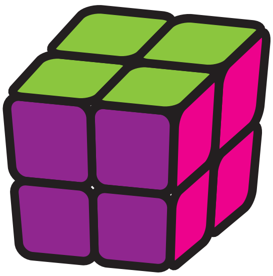
2.2 Solving Four Corners



2.3 Position the Remaining Corners

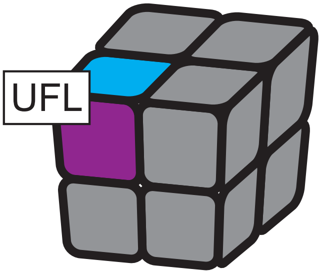


2.4 Orient the Remaining Corners



2.1 Solving Two Corners

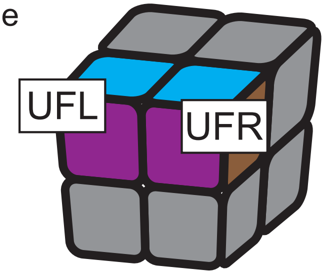
This should be easy (especially if you can solve the 3×3×3 cube), but if you're having trouble getting started, try this.

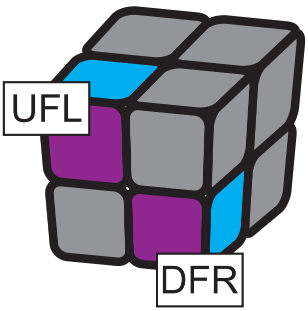


Hold the cube in any position, and look at the UFL (Up/Front/Left) cubie. Let's just say that this corner is correct, and everything else has to match it.

Specifically, we now have a U color (cyan in the example), and an F color (purple in the example).

Hunt around for another cubie that also has the U color and the F color. (Don't lose your UFL cubie, though!) This other cubie will need to go to the UFR position.





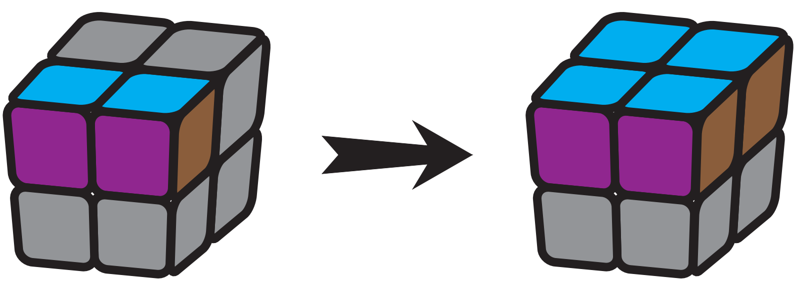
If it's not already there, you can make partial progress by getting it the DFR position. You should be able to do this without disturbing the UFL cube, by simply turning the B (back) face and D (down) face as needed. (If it's on the D layer, just turn the D layer; otherwise, turn the B layer 180° and it will go on the D layer, then turn the D layer as needed.)

Now go to section 3.1.1, and follow those instructions to solve the cubie. If you've read that already and just want a refresher, the basic gist is to repeat sequence A_1 until you're done:

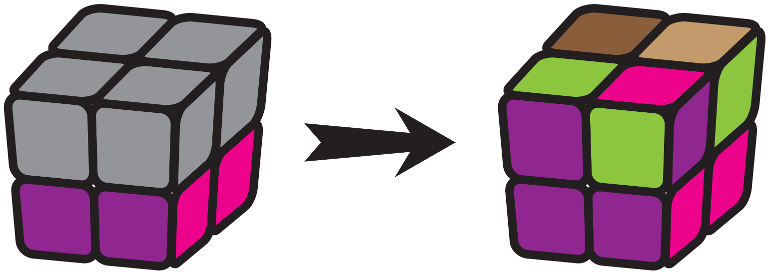
$$A_1 = R\downarrow\ D\leftarrow\ R\uparrow\ D\rightarrow$$

Two corners are now solved. Easy!

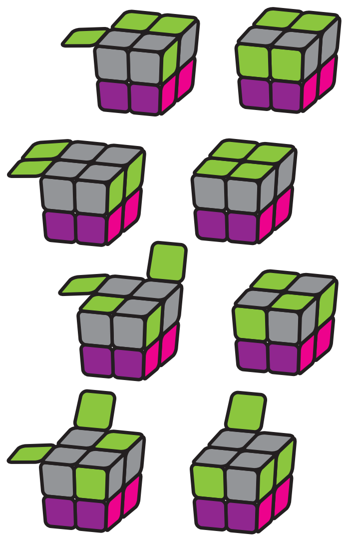
2.2 Solving Four Corners



This is pretty much identical to section 3.1.2. Turn the U layer so that a new UFR location needs to be solved, get the appropriate cubie into the DFR position, and repeat sequence A_1. Do the same for the fourth corner.



Before starting this step, turn over your Cube, so that the layer you've solved is now the D (Down) layer. This part is pretty much the same as section 3.3.1, except that's it's not as obvious what color the U face is suppoed to be.



The secret is that the U face color is the only color that is common to all four unsolved corners. The arrangement should be similar to one of those depicted here on the right (and to the equivalent pictures in section 3.3.1).

Knowing what the U color is, you should have no problem following the rest of the instructions in section 3.3.1 -- find two adjacent cubies with a shared (non-U) color; match them with the D layer, then use sequence C to swap them if necessary. Repeat with the other two cubies.

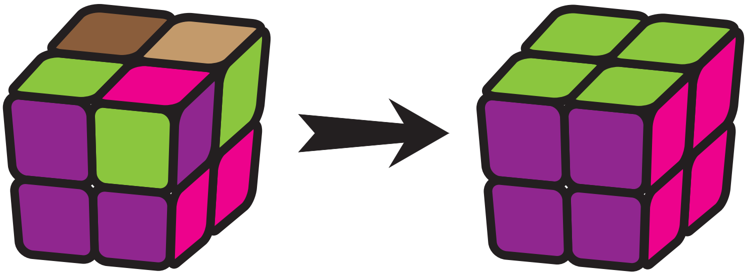
C = F↻ U↻ F↺ U↺ R↑ U↺ R↓

All cubies are now positioned correctly. Now onto orientation!

Advanced Notes: On the 3×3×3, C was one of the most "destructive" sequences; as in, it disturbed the most other cubies for its length. Here, though, it only interferes with the UBL cubie and mixes up some orientation. If you want a sequence that doesn't affect orientation, it'll be twice as long:

F↻ U↻ F↺ U↺ F↺ R↓ F↺ U↻ F↺ U↺ F↺↺ R↑

2.4 Orient the Remaining Corners



This part is pretty much same as 3.3.2; twist the UFR corner to its correct orientation by repeating sequence A_2, then turn the U layer to put a new cubie in the UFR corner, repeat.

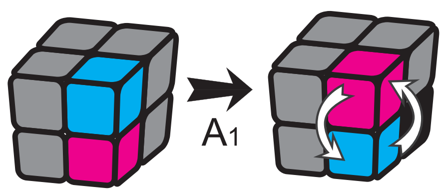
A_2 = R↓ D← R↑ D→ R↓ D← R↑ D→

And the cube is solved!

Here's a review of all the sequences used in the solution for the 2×2×2 cube. Most of these sequences will mess up sections of the cube; we've colored those sections in black.

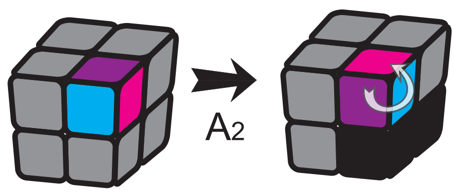
A_1 = R↓ D← R↑ D→

Also messes up the DLB cubie (not shown).

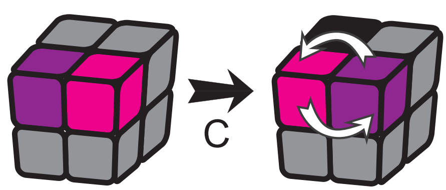


A_2 = R↓ D← R↑ D→
R↓ D← R↑ D→

Repeating A_2 three times will restore what it messed up.



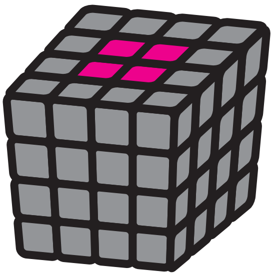
C = F↻ U↻ F↺ U↺
R↑ U↺ R↓



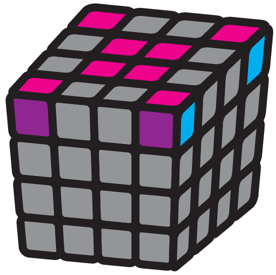
How to Solve the 4×4×4 Cube

Our method of approach will be as follows:

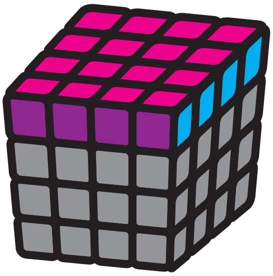
4.1 Solve the U Face Cubies



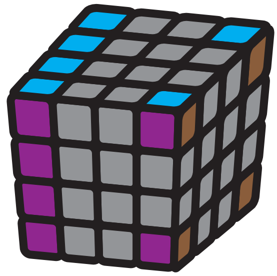
4.2 Solve the U Curner Cubies



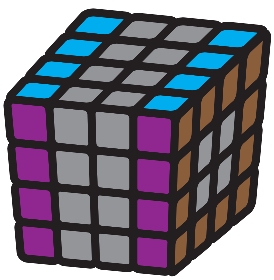
4.3 Solve the U Edge Cubies



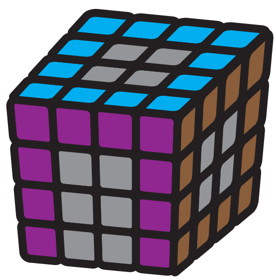
4.4 Solve the Remaining Corners



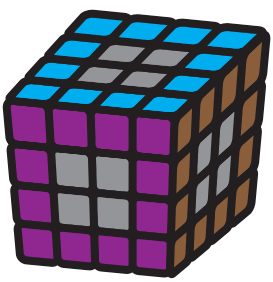
4.5 Solve the R Edge Cubies



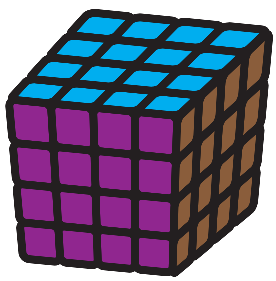
4.6 Solve Two Full Edges



4.7 Solve Remaining Edges



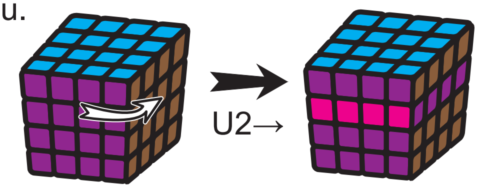
4.8 Solve Remaining Centers



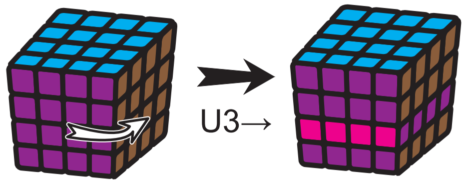
4.0 On Notation

The solution for the 4×4×4 cube assumes that you're already read our solution to 3×3×3 and understand our notation. The only new notation we need to deal with is that now there are twice as many slices. We'll show diagrams for how to deal with slices parallel to the U face; the equivalents for the other directions should be no problem for you.

U2→: Move the second layer (slice) from the top towards the right.

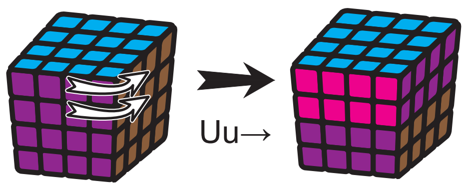


U3→: Move the third layer (slice) from the top towards the right.

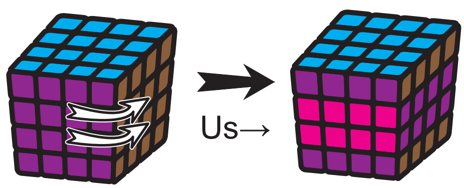


U2 and D3 are equivalent, and same for lots of other pairs (e.g., L2 and R3). Which one we use in describing moves will be based on which one seems easier for you to remember.

Uu→: Move the first *and* the second layer from the top towards the right. (U+U2)



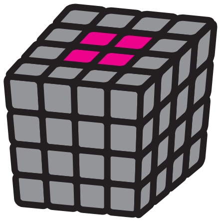
Us→: Move the second *and* the third layer from the top towards the right. (U2+U3)



The reason we now reuse Us to refer to *two* slices is that in the 4×4×4, as long as you always keep the middle two slices together, the cube becomes equivalent to a 3×3×3, albeit with a visually obese middle layer. This means all of the old sequences we learned earlier will still work fine, as long as we know we're moving two edges together. (And although we won't use it, moving the top three layers together to the right would be Uuu→.)

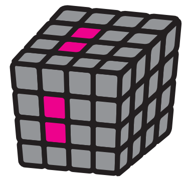
And now, on to the solution!

4.1 Solve the U Face Cubies



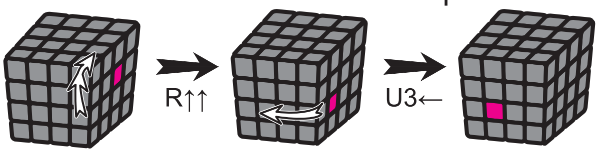
Since the 4×4×4 cube doesn't have a center face cubie, let's start "making" one by matching four identically-colored face cubies together. This should be pretty easy. One way to do it is as follows:

Step 1. Get a pair of adjacent face cubies in the U face.



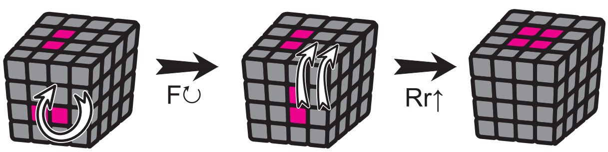
Step 2. Get a matching pair of adjacent face cubies on the F face (without disturbing the U face cubies you already have).

To get a face cubie from the R, L, or B face into the lower-left corner of the F face, turn the face it is on until the cubie is in the lower-left, then turn U3 until it is on the F face. Example:

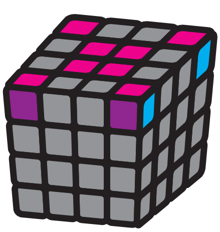


You can use a similar move with U2 instead of U3 if you need the upper-left corner of the F face, or R2 if you need to get the cubie from the U or D face.

Step 3. Turn the U and F faces so that the U cubies are on R3 and the F cubies are on R2. Then do Rr↑ to bring them all together.



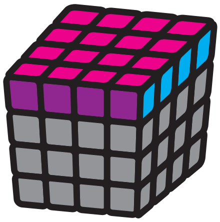
4.2 Solve the U Corner Cubies



This is pretty much identical to section 3.1.2. Turn the U layer so that a new UFR location needs to be solved, get the appropriate cubie into the DFR position, and repeat sequence A_1. Do the same for the fourth corner.

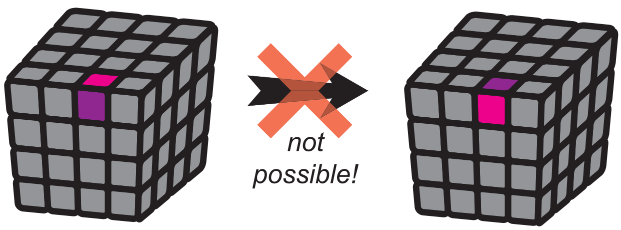
A_1 = R↓ D← R↑ D→

4.3 Solve the U Edge Cubies

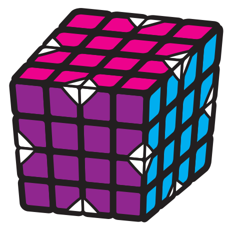


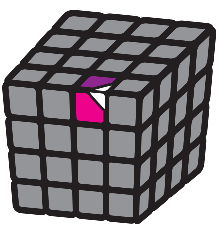
For the most part, we're going to use a similar technique to that of section 3.2, but before we go into that there is a small caveat special to the larger cubes that is worth emphasizing:

It is physically impossible to "flip" a side edge cubie!



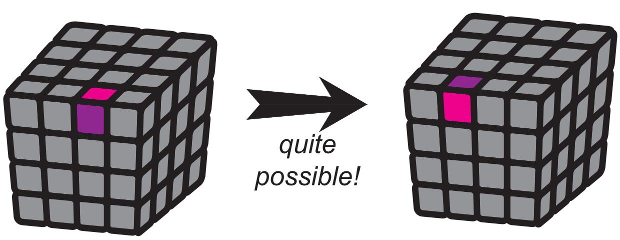
The reason for this is that although the side edge cubies may *look* symmetric, they are internally *not* symmetric. One side of them is closer to the center and it will always stay closer to the center no matter how you manipulate the cube. It might help to imagine a mark at the center of each edge:





If flipping a side edge cubie were possible, then its mark would have to somehow get to the side towards the corner -- but you can see for yourself how no move will ever change where the marks are. In other words, side edge cubies have a **chirality**.

If you see an edge cubie on the 4×4×4 that looks "flipped," it is merely in the wrong location and needs to be in the other spot on the same edge.

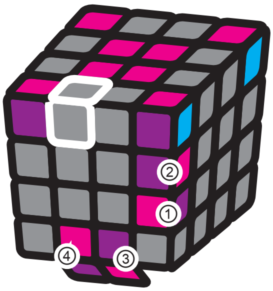


There are many sequences that will do this. One of the simplest ones is U↺R↓F↺.

In any case, our solving technique is the same as in section 3.2:

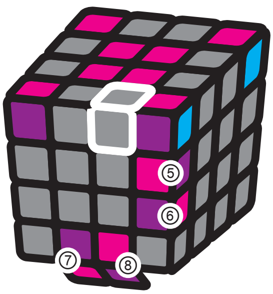
1. Turn the U2, U3, or D layer until the edge piece you want is on the FR or FD edge;
2. Turn the U layer until the “hole” you want is on the UF edge;
3. Use a four-move sequence to bring the edge into the “hole.”

Here are the sequences you’ll need:



When the destination is the left edge cubie of the UF edge:

① FR, lower : U3→ F↻ U3←← F↺
② FR, upper : U2→ F↺ U2← F↻
③ FD, right : F↻ R↑↑ U3→ R↑↑ F↻
④ FD, left : R3↑ F↺ R3↓ F↻



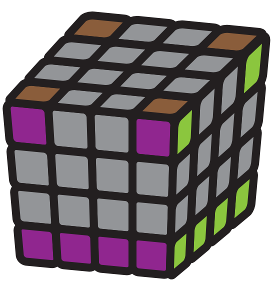
When the destination is the right edge cubie of the UF edge:

⑤ FR, lower : U2→ F↻ U2←← F↺
⑥ FR, upper : U3→ F↺ U3← F↻
⑦ FD, right : F↻ R↑↑ U2→ R↑↑ F↻
⑧ FD, left : R2↑ F↺ R2↓ F↻

(Careful readers will notice the similarities between these moves and those in section 3.2, although ③ ⑦ have been changed since the section 3.2 version disturbs the other UF edge cubie.)

After repeating this (at most) 8 times, you should have the entire U layer solved!

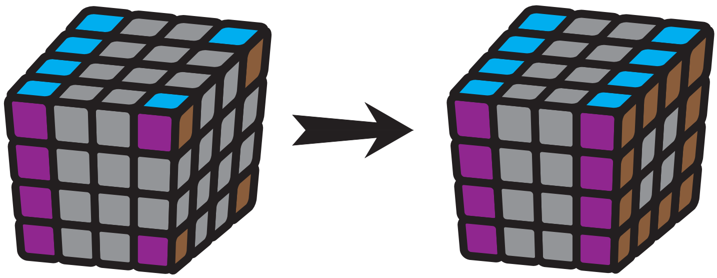
4.4 Solve the Remaining Corners



This part is the same as section 3.3 (with similar caveats as in section 2.3, since there is no center face cubie to guide you). Use sequence C to swap corners and A_2 to twist:

C = F↻ U↻ F↺ U↺ R↑ U↺ R↓
A_2 = R↓ D← R↑ D→ R↓ D← R↑ D→

4.5 Solve the R Edge Cubies



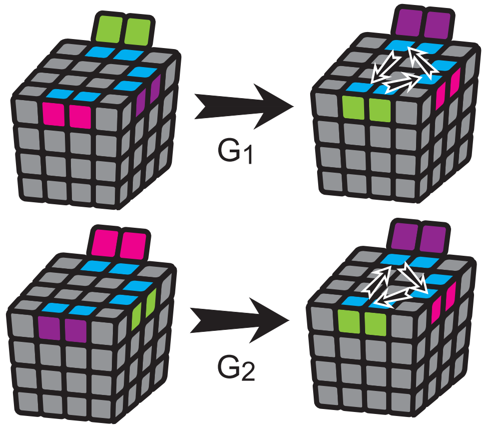
Before starting this, reorient your Cube so that the solved layer is the L layer.

We're going to solve these edge cubies by revisiting our old friends from section 3.4:

G_1 = R↓↓ U↺ Fs↺
U↺↺ Fs↻ U↺ R↓↓

G_2 = R↓↓ U↻ Fs↺
U↻↻ Fs↻ U↻ R↓↓

Don't forget that Fs means *both* middle slices (F2+F3)!

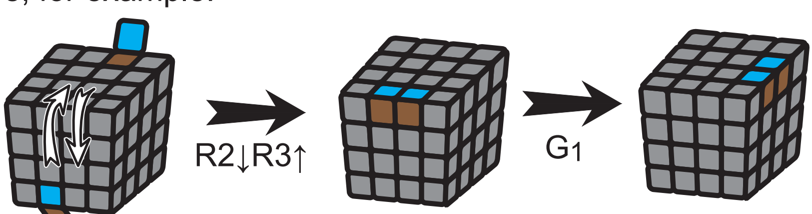


Advanced Notes: Tired of Fs moves? Read the "Advanced Notes" box at the end of section 3.4.

These sequences allow us to move a pair of edge cubies that are on the R2 and R3 slices into the appropriate position onto the R face layer. The only tricky part is getting the cubies to line up.

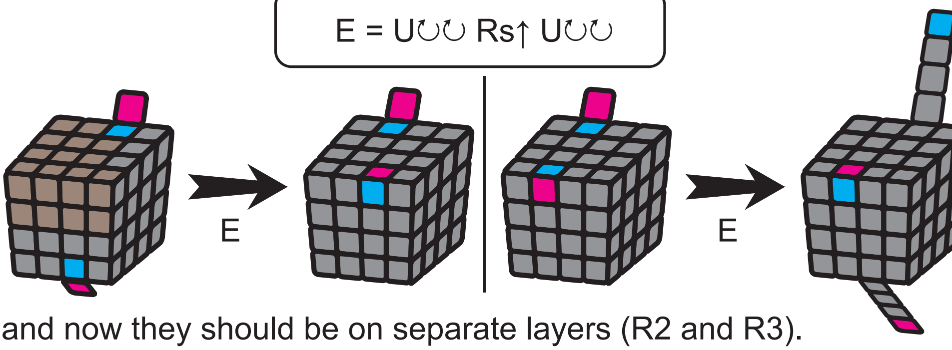
Pick any edge on the R layer and look for the two edge cubies that need to go into that edge.

If the two edge cubies are on the R2 and R3 layer, great! You should be able to turn R2 and R3 to get them in position for a G move, for example:



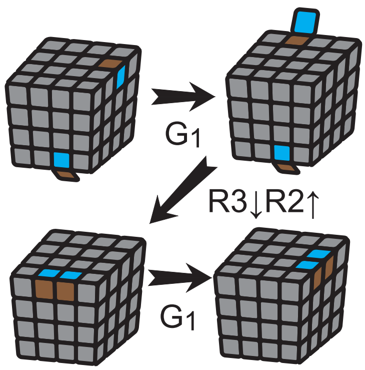
If the two edge cubies are on the same layer (R2 or R3), then turn that layer until both of them are on the U layer (if possible) or just one of them is (if it’s not possible to get both of them on the U layer). Then do

E = U↻↻ Rs↑ U↻↻



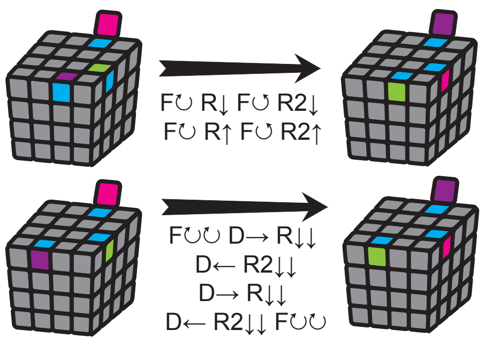
and now they should be on separate layers (R2 and R3). After that, you can pair them up as before.

Finally, if the edge cubies you want are on the R layer, then you’ll need to use G_1 or G_2 to get them off of the R layer and onto R2 or R3, for example:



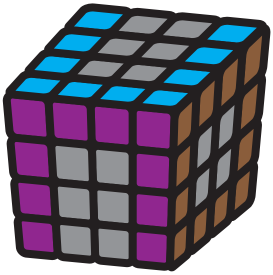
By judiciously repeating this process (getting two edges to match on the R2 and R3 layer, then using a G sequence as appropriate), you should be able to solve all the edges on the R layer.

Advanced Notes:
If you would like to explore moving single edge cubies independently, here are two sequences that move three individual edge cubies on the U layer (without disturbing anything else). The first one is shorter, but the second one only moves edge cubies of the same chirality (keeping the U color on top).



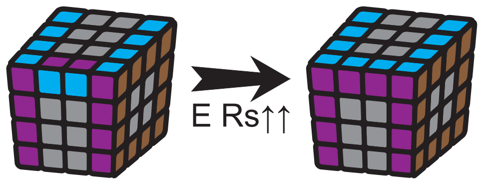
Figuring out *why* these two sequences work is outside the scope of this section, but it’s worth trying to do on your own so you can make similar sequences!

4.6 Solve Two Full Edges

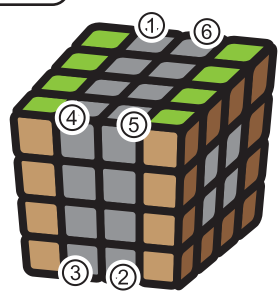


If you've mastered the last section, getting one more edge solved should be pretty simple. Choose any edge and match up the two edge cubies as per the last section. Then, turn Rs until the edge matches with the L and R layers.

Sometimes the edge you've solved will need to be "flipped." Fix this by putting it in the UF position and doing E Rs↑↑.

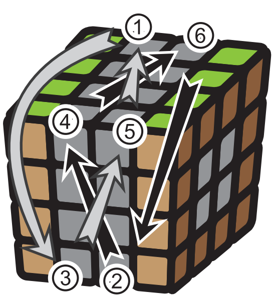


E Rs↑↑ = U↻↻ Rs↑ U↻↻ Rs↑↑



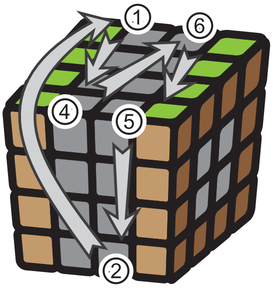
Now that one edge is solved, let's put it out of the way by reorienting the cube so the solved edge is the DB (Down and Back) edge. This leaves the only unsolved edges to be the UB, UF, and DF edges. We'll label them with the numbers ①②③④⑤⑥ as in the diagram to the right, and our next goal will be to get the positions ② and ③ (the DF edge) filled correctly. (The weird numbering is because M_2, on the next page, will fix ②, and M_3 will fix ③.)

If ② is in ③'s position, or vice-versa (you'll be able to tell because it will appear "flipped"), use this sequence M_1 to get it out. (You might even fix the other one!)



M_1 = E Rs↓ = U↻↻ Rs↑ U↻↻ Rs↓

③→⑤→①→③
②→④→⑥→②

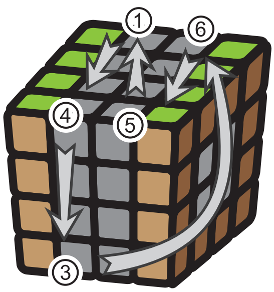


Next, to place ② correctly, use this M_2 (identical to M_1 except that it uses R2 instead of Rs):

M_2 = U↻↻ R2↑ U↻↻ R2↓

②→①→④→⑥→⑤→②

You may need to do this move up to four times, but at least it's easy. (Or you could do anti-M_2 if the cubie is in position ⑤ or ⑥.)



Placing ③ uses (you guessed it) a similar sequence we call M_3:

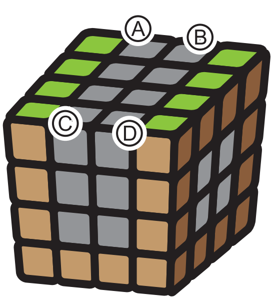
M_3 = U↻↻ R3↑ U↻↻ R3↓

③→⑥→⑤→①→④→③

After this, ② and ③ should be correct, and now there are only four more edge cubies to go.

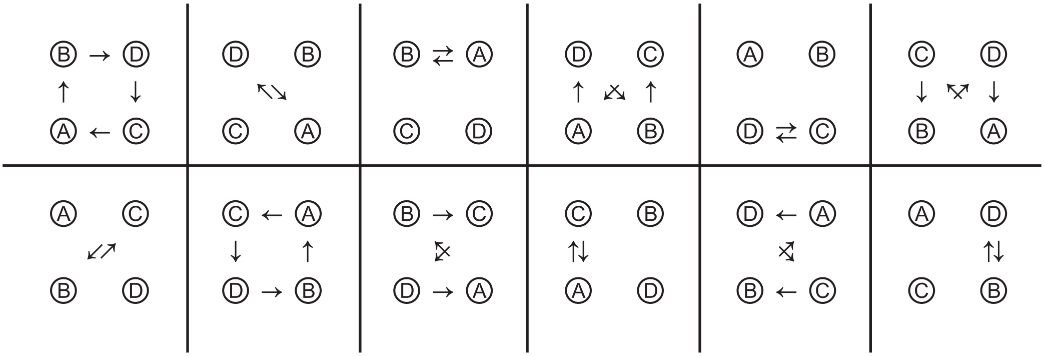
4.7 Solve Remaining Edges

This part is probably the hardest section of the entire solution, as there are 24 ways the four remaining edges can be arranged, and it is not particularly obvious how the solving sequences actually work. Let's start by labelling the four edges ⒶⒷⒸⒹ. (We could have used ①④⑤⑥ from the last section, but it will be a bit less confusing to start afresh.)



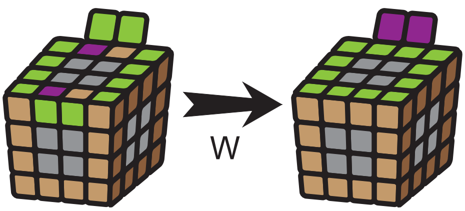
Of the 24 arrangements, 12 of them are "odd" arrangements (they need an odd number of edge swaps to solve, if that makes sense to you), and 12 of them are "even" arrangements. Your first task is to figure out whether your given arrangement is "odd" or "even."

The following 12 arrangements are “odd.” You can tell because they either have a single swap or a length-four cycle:



(Each arrow indicates where the cubie needs to go to, not where it came from.)

If your arrangement is “odd,” you need to make it “even” by using the following move:

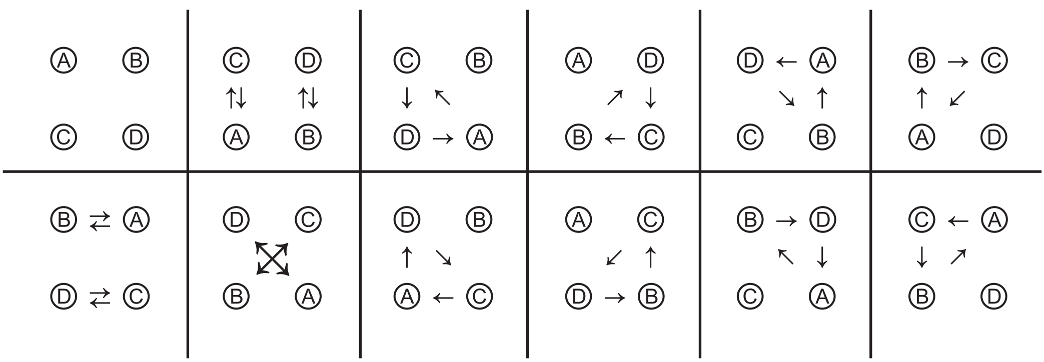


W = Rr↓↓ Uu↻↻
R2↓ Uu↻↻ Rr↓↓
Ⓐ→Ⓑ→Ⓓ→Ⓒ→Ⓐ

(Remember: “Uu” and “Rr” mean to turn two layers together as a group.)

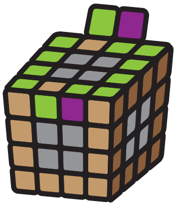
(An alternate version of W, R2↓↓ U↻↻ R2↓ U↻↻ R2↓↓, is easier to memorize but has more slices. Use whichever you like.)

This turns them into one of these “even” arrangements:



The first arrangement in the upper-left is simply solved, so we need to only supply answers for the other 11. Of the 11, eight of them (the ones on the right) are all three-cubie cycles, and they can all be solved by variations on the same sequence:

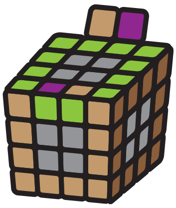
This is the basic sequence for a three-cycle.



Ⓒ   Ⓑ
↓ ↖
Ⓓ → Ⓐ

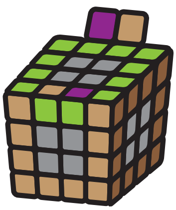
T_1 = Rr↓ D→
R3↓ U↺ R↓↓ U↻
R3↑ U↺ R↑↑ U↻
D← Rr↑

This comes in three more varieties based on Left-Right symmetry and inversion:



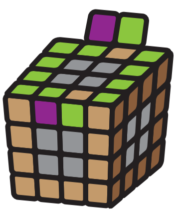
Ⓓ   Ⓑ
↑ ↘
Ⓐ ← Ⓒ

T_2 = Rr↓ D→
U↺ R↓↓ U↻ R3↓
U↺ R↑↑ U↻ R3↑
D← Rr↑



Ⓐ   Ⓒ
↙ ↑
Ⓓ → Ⓑ

T_3 = Ll↓ D→
R2↓ U↺ R↓↓ U↻
R2↑ U↺ R↑↑ U↻
D← Ll↑



Ⓐ   Ⓓ
↗ ↓
Ⓑ ← Ⓒ

T_4 = Ll↓ D→
U↺ R↓↓ U↻ R2↓
U↺ R↑↑ U↻ R2↑
D← Ll↑

Advanced Notes: These sequences use the concept of conjugation, which can be thought of as "taking pieces temporarily to locations where the action is and then bringing them back."

Here, the first two moves bring one of the UF edges to the DR position, the next eight moves cycle three edges, and then the last two moves bring the DR edge (now a different cubie) back to the UF position.

Ⓓ ← Ⓐ
↘ ↑
Ⓒ   Ⓑ

Ⓒ ← Ⓐ
↓ ↗
Ⓑ   Ⓓ

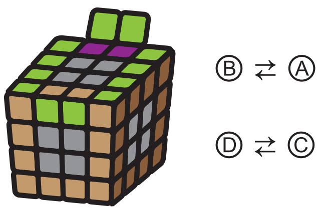
Ⓑ → Ⓓ
↖ ↓
Ⓒ   Ⓐ

Ⓑ → Ⓒ
↑ ↙
Ⓐ   Ⓓ

(The symbol "Ll" is just an "L" followed by a lower-case "L" -- in other words, turn both the L and L2 layers together.)

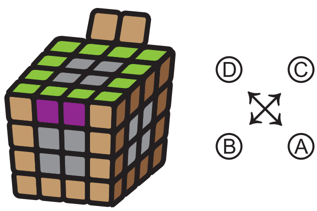
For the other four "three-cycle" arrangements, simply turn the cube 180° around the U face, so that F and B are switched, as well as R and L. Then apply the appropriate T sequence, above.

The other three arrangments (the “double swaps”) are best solved by learning the three specialized sequences here, all of which are pretty simple (and one you should already know). (Alternatively, you can just use any of the T sequences on the previous page, which will fix one of the four cubies. Then choose the appropriate T sequence again to fix it.)



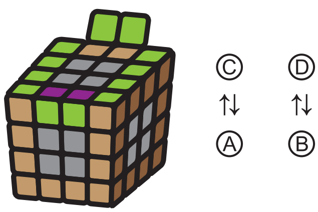
This is the easiest as it is the equivalent of “two edge flips” as covered in seciton 3.6. To refresh your memory, the solution from there is:

V_1 = K_1 Rs↑ K_2 Rs↓
= F↻ U↺ R↑ F↺ U↻ Rs↑
U↺ F↻ R↓ U↻ F↺ Rs↓



This sequence is equivalent to WW (the 180° moves cancel out):

V_2 = Rr↓↓ Uu↻↻
R2↓↓ Uu↻↻ Rr↓↓



You can either do V_1V_2, or this shorter sequence:

V_3 = F↻↻ R2↓ F↻↻ R2↑
R3↓ F↻↻ R3↑ F↻↻

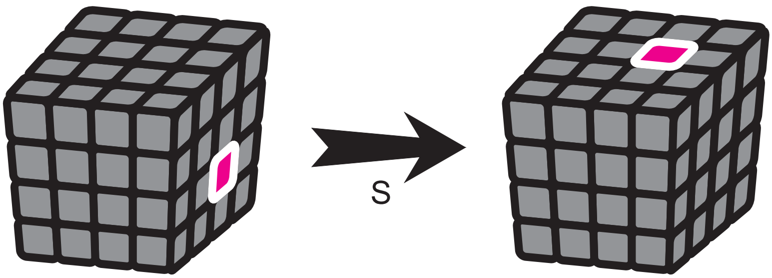
Now all the edges are solved!

Advanced Notes: V_2 and V_3 have a small flaw that you probably won’t care about unless you’re devising your own system -- they disturb the arrangement of some face cubies. If you ever decide to use a method where the face cubies are solved first, you might be desirous of equivalents for V_2 and V_3 that don’t have this side effect. Here they are:

V_2 (alternate) = Ff↻↻ U↻↻ Ff↻ U↻↻ Ff↻↻
U↻↻ Ff↻↻ U↻↻ Ff↻ U↻↻ Ff↻↻
V_3 (alternate) = F↺ U↻ R↑ U3←← R↓ U↺ F↻ U↻↻
F↺ U↻ R↑ U3←← R↓ U↺ F↻ U↻↻

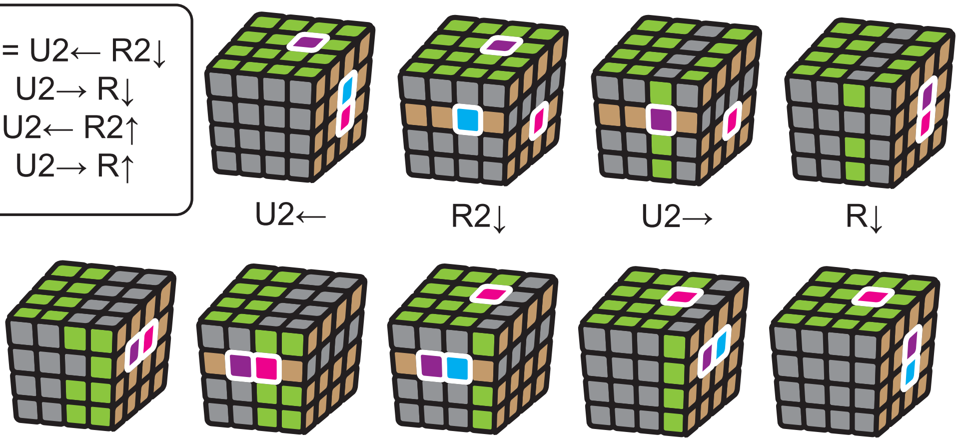
4.8 Solve Remaining Centers

This final section is easier than the others. What we need is a sequence that moves a center cubie from the lower-right (actually DownBack) corner of the Right face to the upper-right (actually RightBack) corner of the Up face:



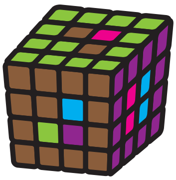
The basic sequence we will use to do this moves three center cubies around in a cycle. (As with many things on the Cubes, there is no “true” way to just swap two face pieces.)

S = U2← R2↓
U2→ R↓
U2← R2↑
U2→ R↑

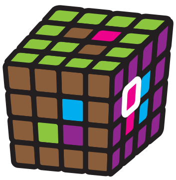


Since we were careful to solve the L face cubies way back in 4.1, we can solve all the face cubies by keeping the R face right and moving different faces to the U layer. The general procedure is:

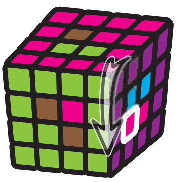
1. Find a misplaced cubie in the R face.
2. Turn the cube so that the destination is in the U face.
3. Turn the R face so that the misplaced cubie is DownBack.
4. Turn the U face so that a destination position is RightBack.
5. Sequence S.
6. Undo the move in step 4.
7. Undo the move in step 3.
8. Repeat from step 1.



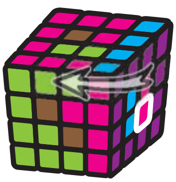
) L G D LWV SODFHG
F ELH L WH 5 IDFH



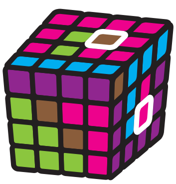
7 U WH EHWR
WDWWH GHWL DWLR
LV L WH 8 IDFH



7 U WH 5 IDFH WR
WDWWH LWV SODFHG
F ELH LV R DFN

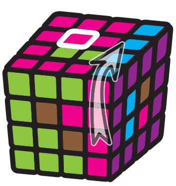


7 U WH 8 IDFH
WR WDWD GHWL DWLR
SRLVWLR LV 5 L WDFN

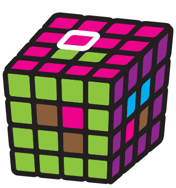


SSO 6HT H FH6
(If you want, you may want to test out steps 6 and 7 before doing S.)

8 GR WH RYH
L WHS
(Note that this is *before* the reversal of step 3, which is obvious if you think about it.)



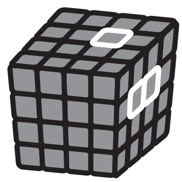
8 GR WH RYH
L WHS



Advanced Notes:
Steps 3-4 / Steps 6-7 are more conjugation!

Advanced Notes:
Sequence S is a very adapatable sequence, with lots of possible tweaks to customize what the sequence does. For example, change the direction of the R moves:

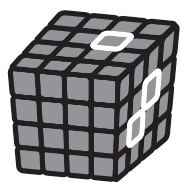
U2← R2↓ U2→ R↑
U2← R2↑ U2→ R↓



and now the UpFront face cubie on the Right face becomes the moving cubie.

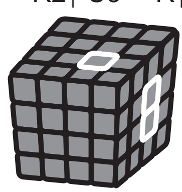
Change R to 180°:

U2← R2↓ U2→ R↑↑
U2← R2↑ U2→ R↓↓



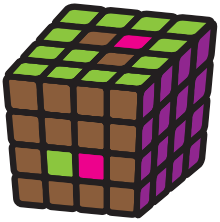
Changing the layer also can work:

U3← R2↓ U3→ R↓
U3← R2↑ U3→ R↑

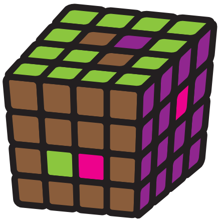


Experiment with this!

Occasionally you will reach a situation where all the face cubies in the R face are solved, but some of the other face cubies still need work. (You can avoid this for the most part by not choosing destination positions that have the R color). If this occurs, you have to to break up your nice R face; move an R face cubie into one of the incorrect face cubies via sequence S. This should move the incorrect face cubie somewhere on the R face, and the solving can continue. Example:

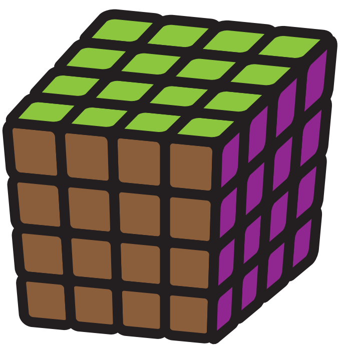


Here the R face is completely solved.



Sequence S pushes an R cube away and a new incorrect cube back onto the R face.

Eventually, you'll place all the face cubies and your Cube is solved!



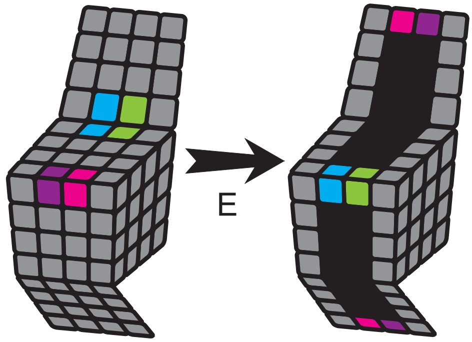
Well done! Give yourself a pat on the back.

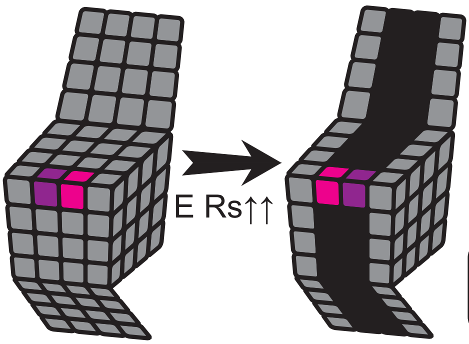
Advanced Notes:
While a "faces last" approach is the simplest to understand, it also tends to be one of the slower approaches, because the face sequences involves lots of slice moves, each of which requires two effective turns. You can save quite a bit of time if you solve the faces first -- but then your edge cubie sequences need to keep the faces untouched, such as the T sequences we've mentioned. More on this in a couple of pages.

Here's a review of all the sequences we've covered that are new to the 4×4×4. The rest are covered in the 3×3×3 section. Again, we color in black the cubies that get hurt as a side effect.

(We won't draw the arrows this time; they criss-cross a lot and get really confusing.)

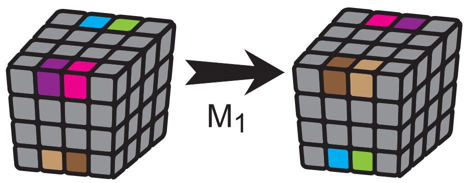
E = U↻↻ Rs↑ U↻↻



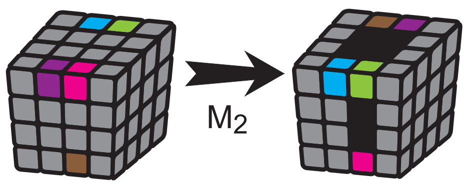


E Rs↑↑ = U↻↻ Rs↑ U↻↻ Rs↑↑

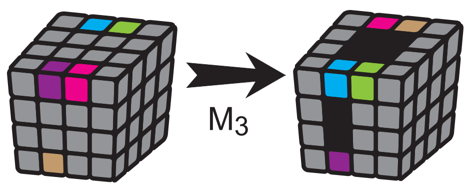
M_1 = E Rs↓
= U↻↻ Rs↑ U↻↻ Rs↓



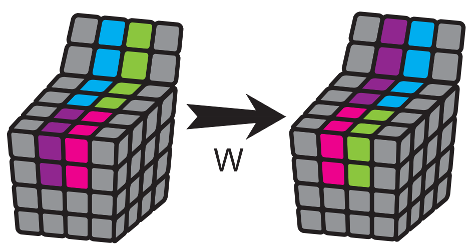
M_2 = U↻↻ R2↑ U↻↻ R2↓



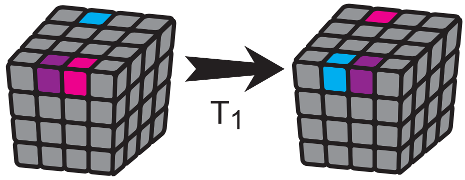
M_3 = U↻↻ R3↑ U↻↻ R3↓



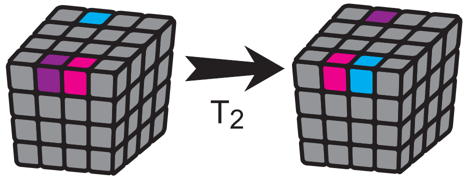
W = Rr↓↓ Uu↻↻
R2↓ Uu↻↻ Rr↓↓



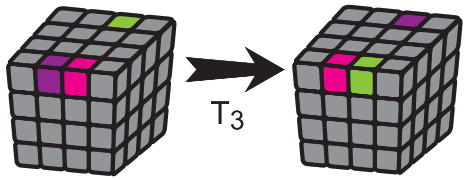
T_1 =
Rr↓ D→
R3↓ U↺ R↓↓ U↻
R3↑ U↺ R↑↑ U↻
D← Rr↑



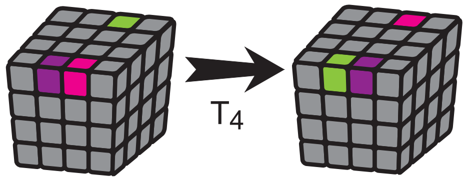
T_2 =
Rr↓ D→
U↺ R↓↓ U↻ R3↓
U↺ R↑↑ U↻ R3↑
D← Rr↑



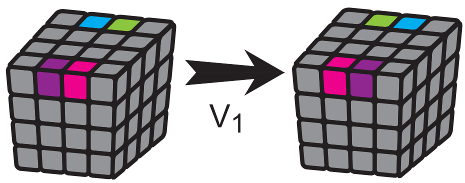
T_3 =
Ll↓ D←
L3↓ U↻ L↓↓ U↺
L3↑ U↻ L↑↑ U↺
D→ Ll↑



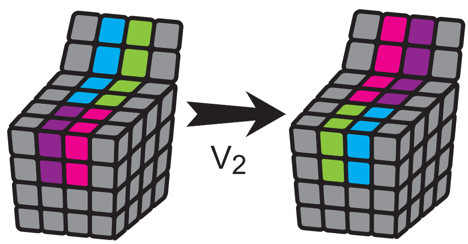
T_4 =
Ll↓ D←
U↻ L↓↓ U↺ L3↓
U↻ L↑↑ U↺ L3↑
D→ Ll↑



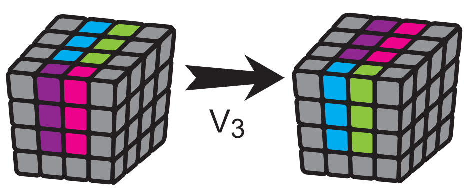
V_1 = K_1 Rs↑ K_2 Rs↓
= F↻ U↺ R↑ F↺ U↻ Rs↑
U↺ F↻ R↓ U↻ F↺ Rs↓



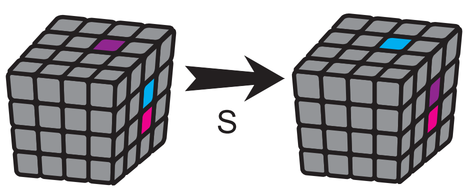
V_2 = Rr↓↓ Uu↻↻
R2↓↓ Uu↻↻ Rr↓↓



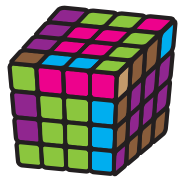
V_3 = F↻↻ R2↓ F↻↻ R2↑
R3↓ F↻↻ R3↑ F↻↻



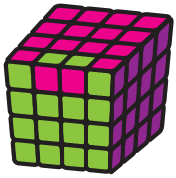
S = U2← R2↓ U2→ R↓
U2← R2↑ U2→ R↑



Bonus Section: Dealing with “Oddness”



People who are used to the 3×3×3 Cube often try to solve the 4×4×4 by trying to reduce it to what they know, by matching the face and edge cubies first. They think that then solving it as a 3×3×3 should be simple.

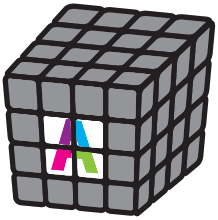


And often, it is. But about half of the time you end up with a situation on the left, where there is an “odd” arrangement (see section 4.7) and no 3×3×3 technique is going to help you.

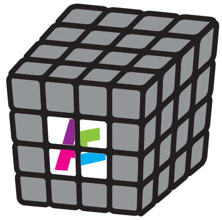
The reason is that the center face cubies are interchangable, creating an effect where you think they are solved, but actually identical-looking pieces need to be swapped. Our method cleverly gets around this by dealing with the issue before the face cubies are solved.

But sometimes that’s not an option. Perhaps your face cubies have pictures and need to be specifically arranged. Or you’re doing a faces-first solution and just want to fix an oddness problem without redoing a lot of face work.

Here’s how to deal with those situations.

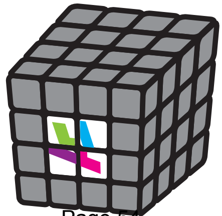


Suppose you’re solving the faces of a cube with pictures (and doing a faces-first system). It should look like that on the left, but two face cubies are swapped, so it looks like that on the right.

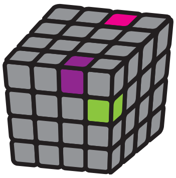


You can do three-cycles on any three faces (because you’re an expert on the many variations of sequence S), but how to handle a single swap?

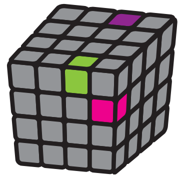
The answer is: you don’t. Turn the face 90°, as in the diagram to the right. Now the swap has turned into a three-cycle, and you can solve that.



If you're solving edges, the simplest three-cycle is as follows:

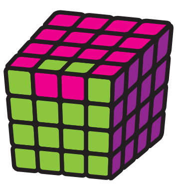


U↺ R↑ U↻ R2↓
U↺ R↓ U↻ R2↑

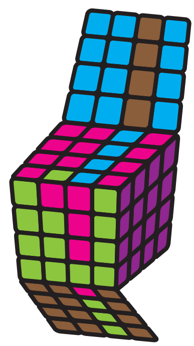


The first three moves put the lime cubie where the purple cubie is without messing up the rest of the R2 layer. Then the R2 layer is moved so that the pink cubie is where the purple cubie started. Moves 5-7 undo the first three moves, so that the purple cubie comes back, and move 8 restores the R2 layer.

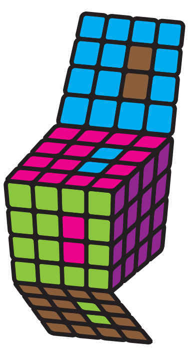
Try to understand how that three-cycle works; it's the fundamental idea behind sequence T, as well as the moves in the "Advanced Notes" in section 4.5. By making adjustments (different R turns, different layers), you can adapt it to cycle any three edge cubies you want.



So what about that two-cycle? No series of three-cycles is ever going to solve this, so what to do?



Simple. Turn R2↓, and now your edges have a five-cycle. That can be fixed with three-cycles.



This whole process basically replaces the oddness problem of edges with an oddness problem in faces -- now we have four sets of faces in a four-cycle. But this is easily fixed:

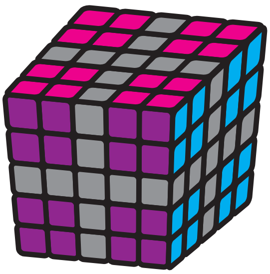
Fs↻ R↓↓ Fs↺ R2↓ Fs↻ R↓↓ Fs↺ R2↓
Fs↻ R↓↓ Fs↺ R2↓ Fs↻ R↓↓ Fs↺ R2↓

(This is actually a five-cycle with pairs of face cubies on the R2 layer and the left pair on the U layer, using the R face as a staging area to swap two such pairs.)

How to Solve the 5×5×5 Cube

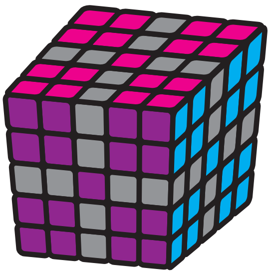
The 5×5×5 Cube, interestingly enough, needs only one new trick.

5.1 Use the 4×4×4 Solution



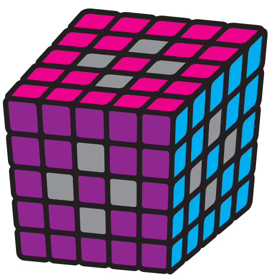
Completely ignore the center slices, and solve the cube as if it were a 4×4×4. (As an extra option, during step 4.1, feel free to solve an entire face, including the center slices on that face -- this will save you some time when you get to 5.4, below.)

5.2 Solve the Center Face Cubes



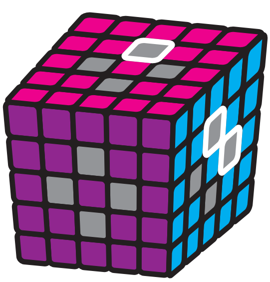
Using center slices, match all the centers (you've probably been tempted to match them already by now anyway). If you need help, check out the first part of section 3.1, where center slice moves are used to move the centers around.

5.3 Use the 3×3×3 Solution to Solve the Center Edge Cubies



Treat the cube as a 3×3×3 with "fat" outside layers, and solve the rest of the edges (ignore the unsolved face cubies for now). You should only need sections 3.2, 3.4, 3.5, and 3.6 for this. (The center edge cubies don't have the "oddness" problem that the 4×4×4 has!)

5.4 Solve the Remaining Face Cubies



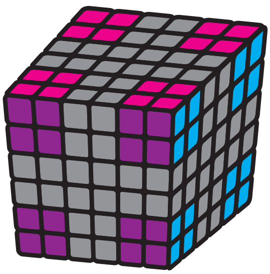
U2← R3↓ U2→ R↓
U2← R3↑ U2→ R↑

All we need here is a variant on the sequence S used in section 4.8; this one moves three face cubies in a cycle (those highlighted at left). By repeating this in the same way as section 4.8, you can solve the rest of the Cube!

How to Solve the 6×6×6 Cube

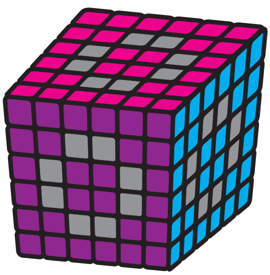
Again, if you can do everything up to this, you'll need very little extra.

6.1 Use the 4×4×4 Solution



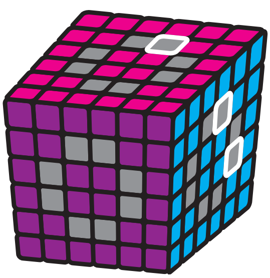
Completely ignore the two center slices, and solve the cube as if it were a 4×4×4.
(As an extra option, during step 4.1, feel free to solve an entire face, including the center slices on that face -- this will save you some time when you get to 6.3, below.)

6.2 Use the 4×4×4 Solution Again



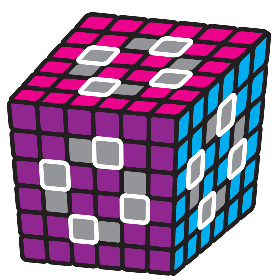
This time, treat the outside layers as "fat" layers, and use the 4×4×4 solution again. Since the "corners" are already solved, you'll only need to deal with the edge sections.

6.3 Use "Sequence S"-like Moves to Solve the Rest



Just as in the 5×5×5, variants on sequence S from section 4.8 can be used to finish the Cube off. The sequence here, for example, cycles the three face cubies highlighted at left.

U2← R3↓ U2→ R↓
U2← R3↑ U2→ R↑



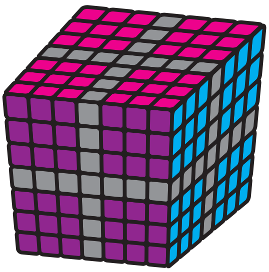
It's worth mentioning that, although they may look interchangable, the remaining 24 face cubies actually come in two chiralities. (Look "chirality" up if you've never encountered the word before.) The "clockwise" ones (highlighted at right) are always going to stay "clockwise" no matter how you move the layers, so don't try to swap them into the "counterclockwise" positions; it won't work!

And with the 6×6×6 solved, we move on...

How to Solve the 7×7×7 Cube

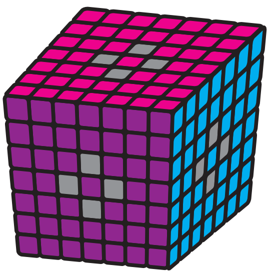
Astute readers can probably write this section all by themselves...

7.1 Use the 6×6×6 Solution



Ignore the middle slices, and...
No surprise here. (If you want, you can skip all parts regarding solving the face cubies, and just deal with all of them together at the end.)

7.2 Use the 5×5×5 Solution



We could have said to use the 3×3×3 solution, but this way we make it one less step since we get one set of face cubies for free. Obviously you'll only need steps 5.2 through 5.4 of the 5×5×5 solution.

7.3 Use "Sequence S"-like Moves to Solve the Rest

Just like section 5.4 again, and the big cube is solved!

Advanced Notes: The reader might be interested in how Wei-Hwa actually solves the large Cubes. His favorite method is based on two considerations: one, that face-cubie sequences are tedious and so the faces should be solved early; and two, that looking for edge cubies is often the slowest part of the solution. Here's his sequence:

1. Face cubies on one face.
2. Face cubies on opposite face.
3. Three edges on a "solved" face and the two corners between them.
4. Same-colored edges and corners on the opposite face.
5. Two adjacent sets of face cubies of two of those side colors.
6. The edge between those two sets.
7. The last two faces.
8. The last four corners.
9. The remaining edge cubies.